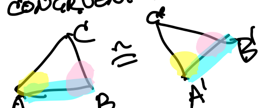
ASA and FTS

Wednesday, September 23, 2020 5:55 PM

LAST TIME: THM (ASA): GIVEN ΔABC AND $\Delta A'B'C'$ WITH $\measuredangle A = \measuredangle A'$, $|AB| = |A'B'|$, AND $\measuredangle B = \measuredangle B'$. THEN $\Delta ABC \cong \Delta A'B'C'$

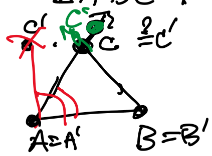
(RECALL: $S \cong R$ IF $\exists$ BASIC ISOMETRY F WITH $F(S) = R$. S CONGRUENT TO R.)



Pf/ FIND F SO THAT $F(\Delta ABC) = \Delta A'B'C'$.

• START

(★) $A = A'$, $B = B'$ AND ASSUME C & C' ARE IN SAME HALF PLANE WRT $\overleftrightarrow{AB}$



SINCE $\measuredangle BAC' = \measuredangle BAC$ HYPOTHESIS

$C' \in \overrightarrow{AC}$ (SINCE ELSE $\measuredangle BAC' \neq \measuredangle BAC$)

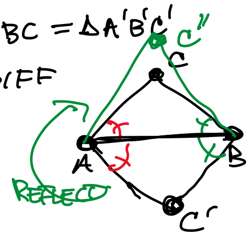
SO $\overrightarrow{AC} = \overrightarrow{AC'}$

BUT ALSO $\measuredangle B = \measuredangle B'$, SO ALSO $\overrightarrow{BC} = \overrightarrow{BC'}$

BUT $C = \overrightarrow{BC} \cap \overrightarrow{AC} = \overrightarrow{BC'} \cap \overrightarrow{AC'} = C'$

SO $C = C'$, SO $\Delta ABC = \Delta A'B'C'$

IF C AND C' ARE IN DIFF HALF PLANES WRT $\overleftrightarrow{AB}$ (BUT $A = A'$, $B = B'$)



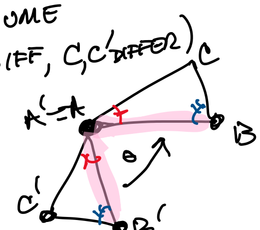
SO USE Λ_{AB}, THEN LET $\Lambda_{AB}(C') = C''$, AND C'' AND C ARE IN THE SAME HALF PLANE

SO USE (★) TO SEE THAT $\Delta ABC \cong \Delta ABC''$ (AND $C = C''$)

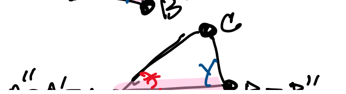
BUT Λ IS A BASIC ISOMETRY SO $\Delta ABC'' = \Lambda(\Delta ABC') \cong \Delta ABC' = \Delta A'B'C'$

NEXT CASE: ASSUME $A = A'$ (B, B' DIFF, C, C' DIFFER)

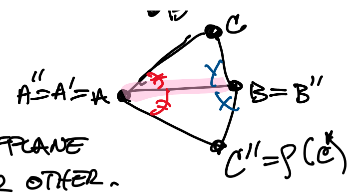
$\measuredangle B'AB = \Theta$ SO I CAN USE $\rho_{\Theta, A}(B') = B'' = B$



AND $\rho(C') = C''$



AND $\rho(C') = C''$



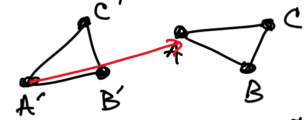
EITHER C'' IS SAME HALFPLANE AS C, OR OTHER.

BUT $A''B'' = AB$, $\measuredangle A = \measuredangle A''$, $\measuredangle B = \measuredangle B''$

WE JUST DID THIS. SO $\Delta A''B''C'' \cong \Delta ABC$
$\rho(\Delta A'B'C') \cong \Delta A'B'C'$. (USING (★) AND MAYBE Λ)

NOW GENERAL CASE WHERE $A \neq A'$, $B \neq B'$, $C \neq C'$

JUST TRANSLATE A' TO A AND THEN ITS THE PREVIOUS CASE.

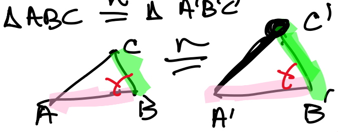


☒

SO ASA CAN BE USED.

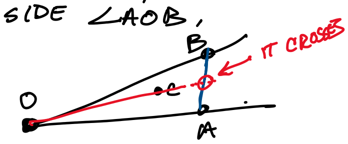
SAS → HW.

THM (SAS): ΔABC & $\Delta A'B'C'$ HAVE $|AB| = |A'B'|$, $\measuredangle B = \measuredangle B'$, $|BC| = |B'C'|$ THEN $\Delta ABC \cong \Delta A'B'C'$



(L8) THE CROSSBAR (THEOREM / AXIOM)

LET $\angle AOB$ BE CONVEX, AND LET $C \neq O$ BE INSIDE $\angle AOB$, THEN $\overrightarrow{OC}$ INTERSECTS $\overline{AB}$



WE HAVE ENOUGH TOOLS TO PROVE THIS. BUT IT IS TECHNICAL AND COMPLICATED USING NUMBERLINES AND ETC. BUT LETS NOT.

NOW TURN TO SIMILARITY FOR A WHILE (THEN COME BACK AND DO THINGS LIKE SSS... STUFF WITH CIRCLES...)

FTS

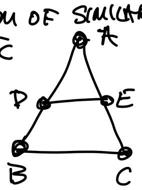
THM G10 (FUNDAMENTAL THEOREM OF SIMILARITY)

GIVEN $\triangle ABC$ WITH D ON $\overline{AB}$, E ON $\overline{AC}$

LET $\dfrac{|AD|}{|AB|} = \dfrac{|AE|}{|AC|} = r$.

THEN $\overleftrightarrow{BC} \parallel \overleftrightarrow{DE}$

AND ALSO $\dfrac{|DE|}{|BC|} = r$



PROOF LATER.

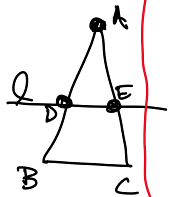
EQUIVALENT VERSION

THM G11 (FTS ALT)

GIVEN $\triangle ABC$ WITH D ON $\overline{AB}$

AND $\ell \parallel \overleftrightarrow{BC}$ WITH $\ell \cap \overleftrightarrow{AC} = E$

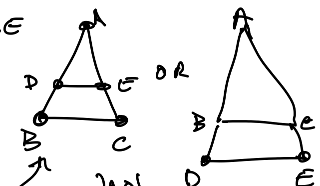
THEN $\dfrac{|AD|}{|AB|} = \dfrac{|AE|}{|AC|} = \dfrac{|DE|}{|BC|}$



G11 $\Leftrightarrow$ G10 SO THESE ARE COMPLETELY EQUIVALENT.

LETS PROVE G11 ASSUMING FTS (G10).

FIGURE COULD BE



IE $\dfrac{|AD|}{|AB|} < 1$ OR $\dfrac{|AD|}{|AB|} > 1$. STILL OK.

LETS PROVE THAT FTS $\Rightarrow$ G11.

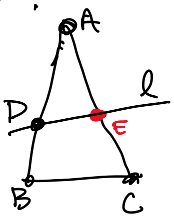
LET ℓ BE PARALLEL TO $\overleftrightarrow{BC}$

LET E BE A POINT ON $\overrightarrow{AC}$

WITH $|AE| = \dfrac{|AD|}{|AB|}|AC|$

$\left(\text{IE } \dfrac{|AE|}{|AC|} = \dfrac{|AD|}{|AB|}\right)$

HAVE TO SHOW $\ell = \overleftrightarrow{DE}$



- USING FTS, $\overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$

SINCE $\overleftrightarrow{DE}$ CONTAINS D AND IS $\parallel$ TO $\overleftrightarrow{BC}$

AND ℓ CONTAINS D AND IS $\parallel \overleftrightarrow{BC}$

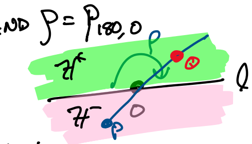
THEY ARE SAME LINE BY PARALLEL POSTULATE.

SO HYPOTHESIS OF FTS ARE SATISFIED SO $\dfrac{|AD|}{|AB|} = \dfrac{|AE|}{|AC|} = \dfrac{|DE|}{|BC|}$ ☒

THM G12 LET $O \in \ell$ AND $\rho = \rho_{180,O}$

THEN ρ SENDS EACH HALFPLANE OF ℓ TO THE OTHER

IE $\rho(\mathcal{H}^+) = \mathcal{H}^-$ AND $\rho(\mathcal{H}^-) = \mathcal{H}^+$.



STEP 1, SHOW THAT $\rho(\mathcal{H}^-) \subseteq \mathcal{H}^+$.

TAKE $P \in \mathcal{H}^-$

LET Q BE ON $\overrightarrow{PO}$ SO THAT $|PO| = |OQ|$

SINCE $\overleftrightarrow{PQ} \cap \ell$ AT O, BY LINE SEP. AXIOM $P * O * Q$ SO $Q \in \mathcal{H}^+$.

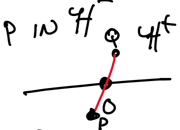
STEP 2 SHOW $\mathcal{H}^+ \subset \rho(\mathcal{H}^-)$.

TAKE $Q \in \mathcal{H}^+$, FIND P IN $\mathcal{H}^-$

SO THAT $\rho(P) = Q$

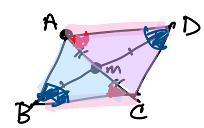
MAKE $\overleftrightarrow{QO}$ FIND P

SO THAT $|OP| = |QO|$. ☺



THM A QUADRILATERAL IS A PARALLELOGRAM

$\Longleftrightarrow$ ITS DIAGONALS BISECT EACH OTHER



PF $\Rightarrow$

HAVE A //OGRAM ABCD.

MUST SHOW THAT $|AM| = |MC|$ AND $|BM| = |MD|$.

SINCE ITS A //OGRAM KNOW OPPOSITE SIDES ARE $\cong$. SO $|AD| = |BC|$ AND $|AB| = |DC|$

WANT TO SHOW $\triangle MAD \cong \triangle MBC$.

SO I NEED TO GET $\angle MAD = \angle MCB$

TO GET THAT, SHOW $\triangle \cong \triangle$ ($\triangle ABC \cong \triangle CDA$)

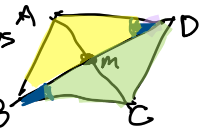
CAN SEE $\angle ADC \cong \angle CBA$ BY ROTATING BY $180°$ AROUND M.

THIS SENDS $C \leftrightarrow A$ AND $B \leftrightarrow D$ (AS IN THE HOMEWORK) SO $\rho(\triangle ABC) = \triangle CDA$

AND $\triangle ABC \cong \triangle CDA$.

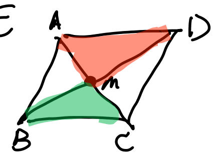
IN PARTICULAR, THIS MEANS $\angle ACB = \angle CAD$

BY A COMPLETELY ANALAGOUS ARGUMENT, WE CAN SHOW $\triangle BAD \cong \triangle DCB$.



AND SO $\measuredangle BDA = \measuredangle DBC$.

FINALLY, WE HAVE



$\angle MDA \cong \angle MBC$

$\angle MAD \cong \angle MCB$

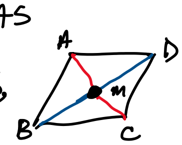
$\overline{AD} \cong \overline{BC}$

SO $\triangle ADM \cong \triangle CBM$

HENCE $|AM| = |MC|$ AND $|BM| = |MD|$. ∎

$\Leftarrow$ THE OTHER DIRECTION IS SHORTER

SPOZE QUADRILATERAL ABCD HAS DIAGONALS MEETING AT M, THE COMMON MIDPOINT OF $\overline{AC}$ AND $\overline{BD}$, THAT IS $|AM| = |MC|$ AND $|BM| = |MD|$. WE HAVE TO SHOW THIS IS A //OGRAM.



BUT A $180°$ ROTATION AROUND M WILL SEND $\overline{AC}$ TO ITSELF, EXCHANGING A AND C, AND $\overline{BD}$ TO ITSELF, AS WELL AS $\angle AMD$ TO $\angle CMB$.

HENCE $\left\{ \begin{array}{l} \overline{AD} \leftrightarrow \overline{BC} \\ \overline{AB} \leftrightarrow \overline{DC} \end{array} \right\}$ BY THIS ROTATION.

SO $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ AND $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$ SINCE A $180°$ ROTATION SENDS LINES TO PARALLELS