Congruences

Monday, September 21, 2020 6:05 PM

LAST TIME: BASIC ISOMETRIES.

(LG) ALL BASIC ISOMETRIES (ROTATIONS, TRANSLATIONS, REFLECTIONS) HAVE THE FOLLOWING PROPERTIES

(i) EACH PRESERVES {LINES}, {RAYS}, {SEGMENTS}, {ANGLES}

(ii) EACH IS AN ISOMETRY (PRESERVES LENGTH OF SEGMENTS) THAT PRESERVES ANGLE MEASURE

DEF A CONGRUENCE IS A TRANSFORMATION OF PLANE WHICH IS A COMPOSITION OF FINITELY MANY BASIC ISOMETRIES.

THM: (a) EVERY CONGRUENCE IS A BIJECTIVE ISOMETRY PRESERVING ANGLE MEAS. PRESERVING {LINES} ..... {ANGLES}.

(b) THE INVERSE OF A CONGRUENCE IS A CONGRUENCE

(c) IF F & G ARE CONGRUENCES, $F \circ G$ IS ALSO A CONGRUENCE.

CONGRUENCE ARE CLOSED UNDER COMPOSITION AND ARE AN EQUIV. RELATION.

Pf/ (a) FOLLOWS FROM AXIOM.

(c) FROM DEF.

(b) TO SEE THAT IF φ IS A CONGRUENCE, THEN φ^{-1} IS TOO...

LET $\varphi = F \circ G \circ H$ SO $\phi = H^{-1} \circ G^{-1} \circ F^{-1}$

SO $\varphi \circ \phi = Id = \phi \circ \varphi$ SO $\varphi^{-1} = \phi$

ALSO, INVERSE OF ROTATION IS ROT, REFLECTION IS REFL, TRANSLATION IS TRANS. ∎

FOR ANY SET S IN THE PLANE, CAN SAY THAT S IS CONGRUENT TO S' IF THERE IS A B.I. φ SO THAT $\varphi(S) = S'$. WRITE $S \cong S'$

CONGRUENT

CONGRUENCE IS AN EQUIVALENCE RELATION.

LEMMA: (a) $|AB| = |CD| \iff \overline{AB} \cong \overline{CD}$

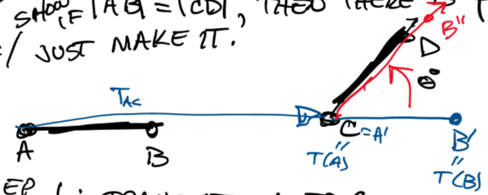
(b) $\measuredangle A = \measuredangle B \iff \angle A \cong \angle B$

NOT DEF. HAVE TO PROVE.

Pf/ (a) $\Leftarrow$ IF $\overline{AB} \cong \overline{CD}$, THEN THERE IS BASIC ISOMETRY $\varphi(\overline{AB}) = \overline{CD}$, AND THIS PRESERVES LENGTH. ∎

$\Rightarrow$ SHOW IF $|AB| = |CD|$, THEN THERE IS φ

Pf/ JUST MAKE IT.



STEP 1: TRANSLATE A TO C

$T_{AC}(\overline{AB}) = \overline{A'B'}$ BUT $A = C$, $= \overline{CB'}$

STEP 2: $\measuredangle B'CD = \theta$

ROTATION $\rho_{\theta, C} = \rho$. $\rho(C) = C$

$\rho(\overrightarrow{CB'}) = \overrightarrow{CB''}$

SINCE $\measuredangle B'CD = \theta$

$\measuredangle B'CB'' = \theta$ BY CONSTRUCTION.

SO $\overrightarrow{CB''} = \overrightarrow{CD}$

AND $|CB''| = |CB'|$ COZ ρ IS ISOM.

$= |AB|$ BECAUSE T ISOM

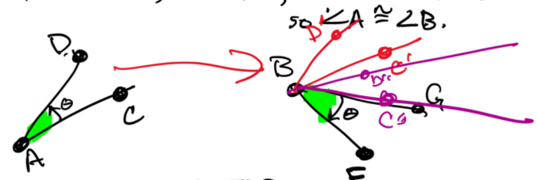
$= |CD|$ BY HYPOTHESIS.

SO $B'' = D$.

SO $\rho \circ T$ IS THE ISOMETRY WE WANT. ∎

(ii) $\angle A \cong \angle B \Rightarrow \measuredangle A = \measuredangle B$ (COZ ISOM) ISOM PRES. ANGLE MEAS.

TO SHOW $\measuredangle A = \measuredangle B$, THEN THERE IS ISOM SO $\angle A \cong \angle B$.



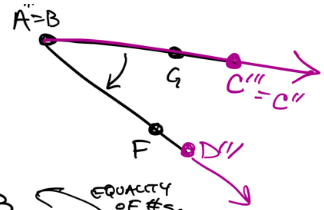
① TRANSLATE A TO B SO $T(A) = B$.

② ROTATE SO THAT $\overrightarrow{BC'} = \overrightarrow{BG}$

$\rho(C') = C''$ WITH C'' ON $\overrightarrow{BG}$

③ IF D'' IS IN SAME HALF PLANE AS F REL $\overleftrightarrow{BG}$, LET $D''' = D''$. IF IN OTHER HALF, THEN APPLY A REFLECTION TO GET $\Lambda(D'') = D'''$

B'' IS ON $\overrightarrow{BF}$ BY UNIQUENESS OF $\angle$ MEASURE



SO $\measuredangle A''' = \measuredangle B$ EQUALITY OF #s.

$\measuredangle A'' \overset{\text{II REFL}}{=} \measuredangle A'' \overset{\text{ROT}}{=} \measuredangle A' \overset{\text{TRANS}}{=} \measuredangle A$ DONE

BASIC ISOM

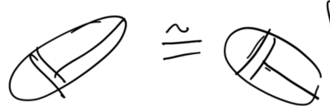
SO ISOMETRY EXISTS. SO $\angle A \cong \angle B$

OFTEN SAY "$\overline{AB}$ IS EQUAL $\overline{CD}$" DOESN'T MEAN EQUALITY OF SETS

MEANS CONGRUENT $|AB| = |CD|$

USUALLY IN GEOMETRY, CONGRUENCE OF GEOMETRIC FIGURES (TRIANGLES, ANGLES POLYGONS, ...

ANY SET WITH (BASIC) ISOMETRIC IMAGE IS CONGRUENT.



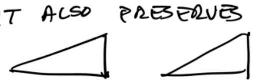
GRAPH OF $y = x^2$ IS CONGRUENT TO GRAPH OF $y = 47 + (x-1)^2$ OR $x = y^2$ ETC...

TRIANGLES ARE ESPECIALLY IMPORTANT IN GEOMETRY. SO WE SPEND A LOT OF TIME ON THEM.

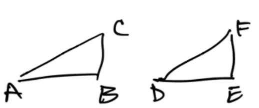
IF I SAY $\triangle ABC \cong \triangle A'B'C'$

NOT ONLY IS THERE AN ISOMETRY φ WITH $\varphi(\triangle ABC) = \triangle A'B'C'$

IT ALSO PRESERVES LABELS.



TRIANGLES ARE CONGRUENT BUT

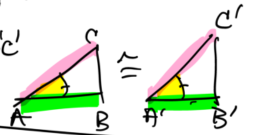


$\triangle ABC \not\cong \triangle EFD$

$\measuredangle A \neq \measuredangle E$

THM (G7) IF FOR $\triangle ABC$ AND $\triangle A'B'C'$ WE HAVE $\measuredangle A = \measuredangle A'$, $\measuredangle B = \measuredangle B'$, $\measuredangle C = \measuredangle C'$ AND $|AB| = |A'B'|$, $|BC| = |B'C'|$, $|CA| = |C'A'|$ THEN $\triangle ABC \cong \triangle A'B'C'$

THM (G8) [SAS] SPOZE IN $\triangle ABC$, AND $\triangle A'B'C'$ WE HAVE $\measuredangle A = \measuredangle A'$, $|AB| = |A'B'|$ AND $|AC| = |A'C'|$ THEN $\triangle ABC \cong \triangle A'B'C'$

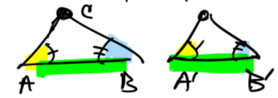


(TWO SIDES & ANGLE BETWEEN MATCH)

THM (G9) ASA IN $\triangle ABC$ AND $\triangle A'B'C'$

$\measuredangle A = \measuredangle A'$, $\measuredangle B = \measuredangle B'$, $|AB| = |A'B'|$

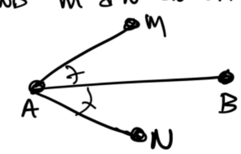
THEN $\triangle ABC \cong \triangle A'B'C'$



LETS PROVE ASA & LEAVE SAS FOR HW.

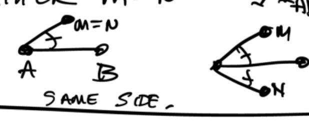
LEMMA: SPOZE CONVEX $\measuredangle MAB = \measuredangle NAB$ SHARE SIDE $\overline{AB}$, AND M & N ON OPP SIDES $\overleftrightarrow{AB}$.

THEN REFLECT. THRU $\overleftrightarrow{AB}$ SENDS $\angle MAB$ TO $\angle NAB$



Pf/ REFLECTION SENDS $\overrightarrow{AM}$ TO $\overrightarrow{AN}$ BECAUSE AXIOM OF UNIQUENESS OF $\angle$ MEAS.

LEMMA SPOZE CONVEX $\angle MAB \cong \angle NAB$ WITH $\overline{AB}$ IN COMMON. AND $\overline{AM} \cong \overline{AN}$. THEN EITHER $M = N$ OR $\Lambda_{AB}(M) = N$.



Pf/ • IF M, N ON SAME SIDE OF $\overleftrightarrow{AB}$ THEN $\overrightarrow{AM} = \overrightarrow{AN}$, SINCE $|AM| = |AN| \Rightarrow M = N$

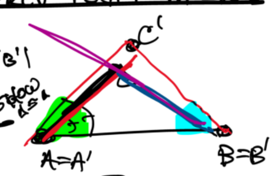
• IF M, N ON OPPOSITE SIDES, $\Lambda_{AB}(M) = M'$ ON SAME SIDE, APPLY PREV TO GET $M' = N$.

ASSUME $|AB| = |A'B'|$

$\measuredangle BAC = \measuredangle B'A'C'$

AND $\measuredangle ABC = \measuredangle A'B'C'$ SHOW ASA

SPECIAL CASE: $A = A'$, $B = B'$



$\overline{AB} = \overline{A'B'}$ AND C, C' IN SAME HALF

MUST SHOW $C = C'$

SINCE $\measuredangle BAC = \measuredangle B'A'C'$ SAME

HAVE $\overrightarrow{AC} = \overrightarrow{A'C'} = \overrightarrow{AC'}$

ALSO $\measuredangle A'B'C' = \measuredangle ABC$ SAME

SO $\overrightarrow{BC} = \overrightarrow{B'C'} = \overrightarrow{BC'}$

TWO RAYS $\overrightarrow{AC} \cap \overrightarrow{BC} = C$

$\overrightarrow{AC'} \cap \overrightarrow{BC'} = C'$

SO $C' = C$.

ISOMETRY IS IDENTITY.

IF C, C' IN ~~DIFF HALF~~.

SUMMARIZE: