Reflections andTranslations

Wednesday, September 16, 2020 5:59 PM

LAST TIME:

- AXIOMS FOR ROTATIONS (ISOMETRY, PRESERVE, SEND LINES→LINES, ETC)
- THM(G1) ℓ LINE, O NOT ON ℓ, $\Rightarrow \rho_{180,O}(\ell) \parallel \ell$
- COR. PLAYFAIR // AXIOM (UNIQUE)
- THM(G2) IF $\ell \perp m$, $\ell_2 \perp m$, $\ell_2 \parallel \ell_1$ OR $\ell_1 = \ell_2$
- THM(G3) SPOZE $\ell_1 \parallel \ell_2$, m TRANSVERSAL $m \perp \ell_1 \Rightarrow m \perp \ell_2$

COR: $P \in \ell$ THEN THERE IS EXACTLY ONE LINE m CONTAINING P WITH $m \perp \ell$

COR: RECTANGLES EXIST.

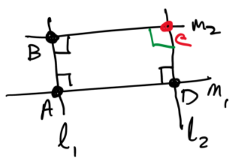
PF/ LET ℓ_1, ℓ_2 BE LINES $\perp$ TO m, AT POINTS A & D

LET $m_2 \perp \ell_1$ AT B.

CALL INTERSECTION OF m_2 AND ℓ_2 C

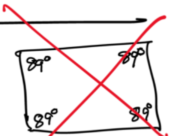
BY (G2) $m_2 \parallel m$,

AND (G3) $\Rightarrow m_2 \perp \ell_2$ ☒



WHAT ABOUT QUAD WITH 4 89° ANGLES?

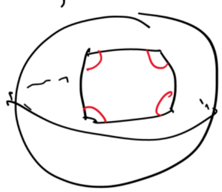
NOPE.



WILL TAKE WORK TO PROVE THIS. (ANGLE SUM IN QUAD IS 360° ≠ Δ=180°)

IN HYPERBOLIC GEOMETRY THERE ARE NO RECTANGLES, BUT EXIST

ON A SPHERE ANGLES > 90° (NO RECTANGLES!)

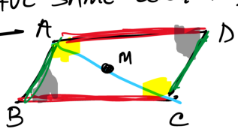


THM (G4) OPPOSITE SIDES OF A PARALLELOGRAM HAVE SAME LENGTH.

GIVEN ▱ABCD //OGRAM

SHOW $|AD| = |BC|$

$|AB| = |CD|$



LET M BE MIDPOINT OF $\overline{AC}$

LET $\rho_{180,M} = \rho$

SO $\rho(C) = A$, $\rho(\overleftrightarrow{BC})$ CONTAINS A

$\rho(\overleftrightarrow{BC}) = \overleftrightarrow{AD}$ $\Big\}$ $\rho(B) = D$

ALSO $\rho(\overleftrightarrow{AB}) = \overleftrightarrow{CD}$

SINCE ρ IS ISOM

SO $\rho(\overline{BC}) = \overline{AD}$, SO $|\overline{AD}| = |\overline{BC}|$

$\rho(\overline{AB}) = \overline{CD}$, SO $|\overline{AB}| = |\overline{CD}|$ ☒

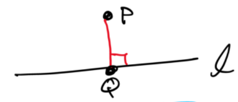
COR. OPPOSITE ANGLES OF A PARALLELOGRAM HAVE SAME MEASURE

SINCE ρ PRESERVES ANGLE MEASURE.

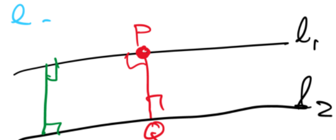
LET P BE A POINT NOT ON ℓ

- DEFINE $\text{dist}(P, \ell) = |\overline{PQ}|$

REALLY THE MINIMAL DISTANCE FROM P TO ℓ.



- IF $\ell_1 \parallel \ell_2$ $\text{dist}(\ell_1, \ell_2)$ = LENGTH OF OF ANY MUTUALLY PERPENDICULAR SEGMENT



REFLECTIONS

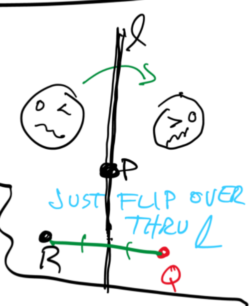
DEF: GIVEN A LINE ℓ, THE REFLECTION Λ_ℓ ACROSS ℓ IS TRANSFORMATION:

① IF $P \in \ell$, $\Lambda_\ell(P) = P$

② IF $R \notin \ell$, $\Lambda_\ell(R) = Q$

WHERE $\text{dist}(R, \ell) = \text{dist}(Q, \ell)$ WITH Q IN OTHER HALFPLANE

Q IS THE POINT SO THAT ℓ IS PERP. BISECTOR OF $\overline{RQ}$

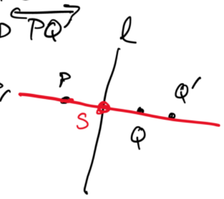


TO SEE THAT Λ_ℓ IS WELL DEFINED!

(IE $\Lambda_\ell(P) = Q$ AND $\Lambda_\ell(P) = Q'$ THEN $Q = Q'$) ← ALWAYS GET SAME OUTPUT

SO SPOZE THERE WERE TWO SUCH Q & Q'. THEN $\overleftrightarrow{PQ}$ AND $\overleftrightarrow{PQ'}$ BOTH $\perp$ TO ℓ

BY (G3) $\overleftrightarrow{PQ} = \overleftrightarrow{PQ'}$



NOTE THAT Q & Q' ARE SAME HALFPLANE WRT ℓ (SINCE NOT IN P'S HALF)

LET S BE $\overleftrightarrow{PQ} \cap \ell$

$|\overline{QS}| = |\overline{Q'S}|$ COZ S IS MIDPOINT OF $\overline{PQ}$ AND ALSO $\overline{PQ'}$

SO $Q = Q'$

NOTICE THAT ANY REFLECTION Λ_ℓ

$\Lambda_\ell \circ \Lambda_\ell = I_d \Rightarrow \Lambda_\ell$ IS BIJECTION. ($\Lambda = \Lambda^{-1}$)

REFLECTIONS ARE BIJECTIONS

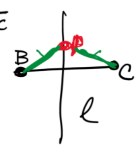
ASSUMPTIONS/AXIOMS ABOUT REFLECTIONS

(Λ_1) EVERY REFLECTION SENDS LINES TO LINES, SEGMENTS TO SEGMENTS RAYS TO RAYS, ANGLES TO ANGLES.

(Λ_2) EVERY REFLECTION PRESERVES LENGTH OF SEGMENTS (ISOMETRY) AND MEASURE OF ANGLES

LEMMA: GIVEN ANY LINE ℓ, THERE IS A REFLECTION Λ_ℓ

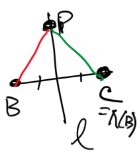
LEMMA: GIVEN A SEGMENT $\overline{BC}$ LET ℓ BE ITS $\perp$ BISECTOR THEN EVERY POINT ON ℓ IS EQUIDISTANT FROM B AND C



LET P BE ANY POINT OF ℓ (MUST SHOW $|\overline{BP}| = |\overline{CP}|$)

CONSIDER $\overline{BP}$, $\Lambda_\ell(\overline{BP}) = \overline{CP}$

BUT Λ PRESERVES LENGTH SO $|BP| = |CP|$

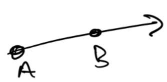


TRANSLATIONS

I'VE BEEN USING $\overrightarrow{AB}$ FOR RAY FROM A THRU B

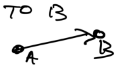
AUTHOR USES R_{AB}

BUT I NEED VECTORS



$\overrightarrow{AB}$ IS NOTATION FOR VECTOR FROM A TO B

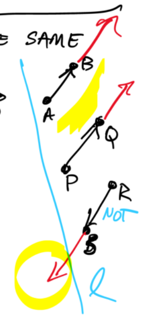
I'LL USE $\langle A,B\rangle$ FOR VECTOR

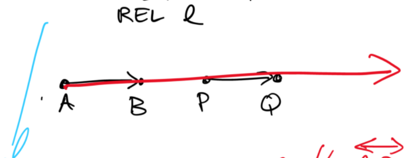


A VECTOR IS A DIRECTED SEGMENT IE DIRECTION + DISTANCE

DEF TWO VECTORS $\langle AB\rangle$ AND $\langle PQ\rangle$ POINT IN THE SAME DIRECTION IF

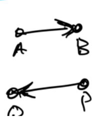
- $\langle AB\rangle = \langle PQ\rangle$ OR $\overleftrightarrow{AB} \parallel \overleftrightarrow{PQ}$
- THERE IS A LINE ℓ DISTINCT FROM $\overleftrightarrow{AB}$ AND $\overleftrightarrow{PQ}$ NOT $\parallel$ TO $\overleftrightarrow{AB}$ SO THAT $\overrightarrow{AB}$ AND $\overrightarrow{PQ}$ LIE IN SAME HALF PLANE REL ℓ





TROUBLE IF $\ell \parallel \overleftrightarrow{AB}$

ℓ



DON'T LIKE.

LEMMA GIVE $\langle AB\rangle$ AND A POINT P, THEN THERE IS A UNIQUE VECTOR $\langle PQ\rangle$ IN THE SAME DIR AS $\langle AB\rangle$ WITH $|\overline{AB}| = |\overline{PQ}|$

DEF: GIVEN $\langle AB\rangle$, THE TRANSLATION ALONG $\langle AB\rangle$ IS THE TRANSFORMATION T_{AB} SO THAT $T_{AB}(P) = Q$ WHERE Q IS THE ENDPOINT OF $\langle PQ\rangle$ WITH $\langle PQ\rangle$ AND $\langle AB\rangle$ IN SAME DIR AND $|PQ| = |AB|$

ASSUMPTION/AXIOMS ABOUT TRANSLATIONS

(T1) ANY TRANSLATION SENDS LINES TO LINES SEGMENTS TO SEGMENTS, RAYS TO RAYS ANGLES TO ANGLES,

(T2) ANY TRANSLATION PRESERVES LENGTH OF SEGMENTS (ISOMETRY) AND ANGLE MEASURE

CALL TRANSFORM THAT IS A REFLECTION, A ROTATION OR A TRANSLATION

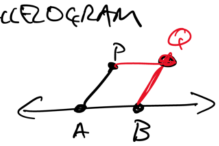
BASIC ISOMETRY

(L7) IF F IS A BASIC ISOMETRY THEN:

(i) F SENDS LINES TO LINES, SEGMENTS TO SEGMENTS, RAYS TO RAYS, ANGLES TO ANGLES.

(ii) F IS AN ISOMETRY THAT PRESERVES ANGLE MEASURE

LEMMA: SPOSE $P \notin \overleftrightarrow{AB}$ AND $T_{AB}(P) = Q$ THEN Q IS THE UNIQUE POINT SO $\square ABQP$ IS A PARALLELOGRAM



THM (G5) LET ℓ BE A LINE NOT PARALLEL TO $\overleftrightarrow{AB}$ AND NOT EQUAL TO $\overleftrightarrow{AB}$ THEN T_{AB} SENDS ℓ TO A LINE PARALLEL TO ℓ.