Circles, regions, & transformations

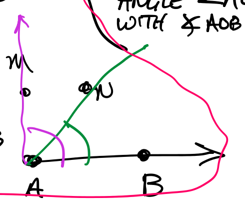
Wednesday, September 9, 2020 5:48 PM

LAST TIME WE ENDED WITH (L6) WHICH DESCRIBES ANGLE MEASURE

IMMEDIATE CONSEQUENCE OF UNIQUENESS PART (ii?) → GIVEN RAY $\overrightarrow{OB}$ AND $0 \le x \le 360$ AND A (CLOSED) HALF PLANE OF $\mathcal{H}$ $\overleftrightarrow{OB}$, THERE IS $A \in \mathcal{H}$ AND ANGLE $\angle AOB$ WITH $\measuredangle AOB = x^0$

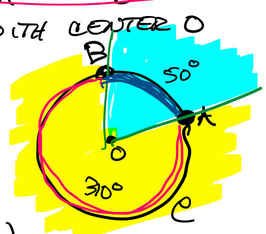
LEMMA: LET $\angle MAB$, $\angle NAB$ BOTH BE CONVEX (OR BOTH NON CONVEX) SHARING $\overrightarrow{AB}$, M & N ON SAME SIDE OF $\overleftrightarrow{AB}$

THEN $\overrightarrow{AM} = \overrightarrow{AN} \iff \measuredangle MAB = \measuredangle NAB$



LET $\mathcal{C}$ BE A CIRCLE WITH CENTER O AND A, B BE POINTS ON $\mathcal{C}$.

$\angle AOB \cap \mathcal{C} = \widehat{AB}$ ← ARC



LETS SAY MEASURE $(\widehat{AB}) = \measuredangle AOB$.

MEAS$(\widehat{AB})$ $<$ OR $\le 180^\circ \Rightarrow$ MINOR ARC

$= 180^\circ \Rightarrow$ SEMICIRCLE

$>$ OR $\ge 180^\circ \Rightarrow$ MAJOR ARC

YOUR TASTE.

ANGLES ACUTE (MEAS $< 90^\circ$)

RIGHT (MEAS $= 90^\circ$)

OBTUSE (MEAS $> 90^\circ$ ... $< 180^\circ$)

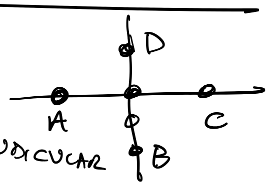
STRAIGHT $= 180$

I WONT USE [REFLEX ANGLE $> 180^\circ$]

DEF IF $\overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \{O\}$ AND $\measuredangle AOB = 90^\circ$

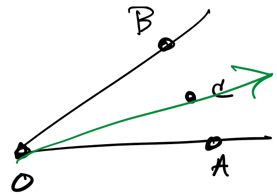
THEN $\overleftrightarrow{AC}$ IS PERPENDICULAR TO $\overleftrightarrow{BD}$

$\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

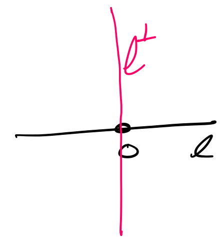


$AC \perp BD$

DEF GIVEN $\angle AOB$, C INTERIOR TO $\angle AOB$, RAY $\overrightarrow{OC}$ IS CALLED THE BISECTOR OF $\angle AOB$ IF $\measuredangle AOC = \measuredangle COB$

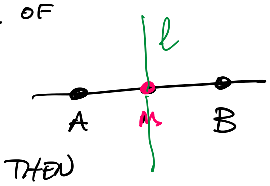


LEMMA H10: LET ℓ BE A LINE CONTAINING O. THEN THERE IS A UNIQUE LINE $\ell^\perp$ CONTAINING O SO THAT $\ell^\perp \perp \ell$



DEF GIVEN $\overrightarrow{AB}$, THERE IS A UNIQUE MIDPOINT M OF $\overline{AB}$ WITH $A * M * B$ AND $|AM| = |MB|$

AND IF $\ell \perp \overleftrightarrow{AB}$ AND $M \in \ell$, THEN ℓ IS THE PERPENDICULAR BISECTOR OF $\overline{AB}$



YOU CAN READ OR FIGURE OUT DEFS OF EQUILATERAL Δ

ISOSCELES

QUADRILATERAL

RECTANGLE

SQUARE

TRAPEZOID

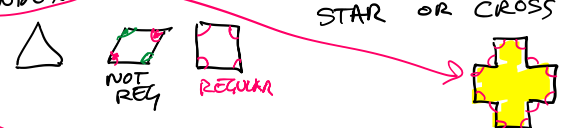
PARALLELOGRAM

RHOMBUS

(SEE P. 251–2 NOTES §4.2)

REG POLYGON
- ALL SIDES = LENGTH
- ANGLES AT VERTICES =
- CONVEX

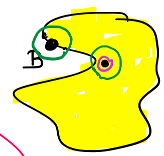
STAR OR CROSS



COULD GET RID OF "INTERIOR ∡ ="

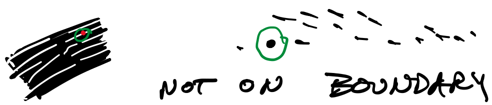
LET S BE A SUBSET OF THE PLANE

DEF: A POINT $B \in S$ IS A BOUNDARY POINT OF S IF



IF "B IS INBETWEEN YELLOW AND WHITE" ← MEANS WHAT?

EVERY DISK WITH B AS CENTER CONTAINS POINTS OF S AND POINTS NOT IN S

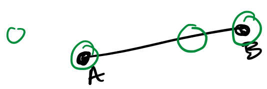


NOT ON BOUNDARY IF I CAN WIGGLE POINT A LITTLE AND STAY IN (OR STAY OUT)

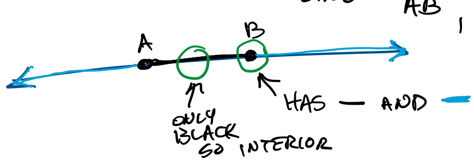
USEFUL NOTATION: ∂S FOR THE BOUNDARY OF S

IDEA OF BOUNDARY DEPENDS ON WHAT IT IS A SUBSET OF.

FOR EXAMPLE A SEGMENT $\overline{AB}$ IN PLANE $\partial(\overline{AB}) = \overline{AB}$



BUT AS A SUBSET OF LINE $\overleftrightarrow{AB}$, $\partial(\overline{AB}) = \{A, B\}$

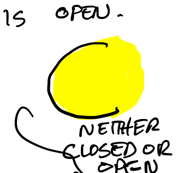


DEF: IF THE BOUNDARY OF S IS CONTAINED IN S, THEN S IS CLOSED

NOTE: NOT CLOSED DOES NOT MEAN OPEN.

IN PLANE: $\partial S \cap S = \emptyset$ IS OPEN.

CAN YOU GIVE AN EXAMPLE OF A SET THAT IS OPEN AND CLOSED?

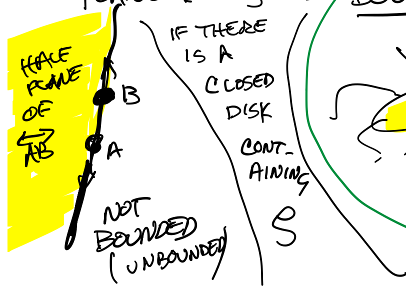


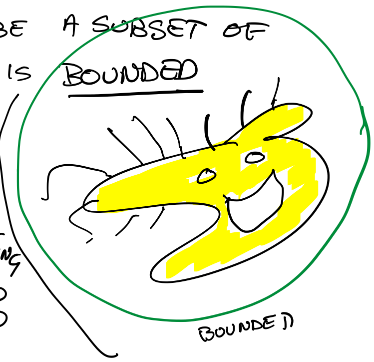
YES: THE PLANE $= S$ $\quad S \cap \emptyset = \emptyset$
IS CLOSED CUZ IT HAS NO BOUNDARY.

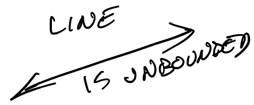
SO IT DOESN'T CONTAIN ITS BOUNDARY.

ALSO $\emptyset$ HAS NO BOUNDARY BUT ALSO NO POINTS
$S \cap \emptyset = \emptyset$

DEF: LET S BE A SUBSET OF PLANE. S IS BOUNDED IF THERE IS A CLOSED DISK CONTAINING S



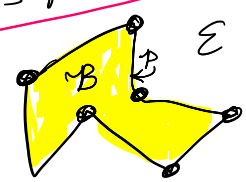




THM: A POLYGON $\mathcal{P}$ SEPARATES THE PLANE INTO TWO NONEMPTY SUBSETS $\mathcal{B}$ AND $\mathcal{E}$ SUCH THAT

- $\mathcal{B} \cup \mathcal{P} \cup \mathcal{E}$ = PLANE
- THESE SETS ARE PAIRWISE DISJOINT } DISJOINT UNION
- $\mathcal{B}$ IS BOUNDED W/ $\partial\mathcal{B} = \mathcal{P}$ (INTERIOR)
- $\mathcal{E}$ IS UNBOUNDED W/ $\partial\mathcal{E} = \mathcal{P}$ (EXTERIOR)
- IF $P \in \mathcal{B}$ AND $Q \in \mathcal{E}$, THEN $\overline{PQ}$ INTERSECTS $\mathcal{P}$

- SIMILAR (L4) PLANE SEP. OR HW1 #8.
- SPECIAL CASE OF JORDAN CURVE THEOREM.



DEF: A POLYGONAL REGION $\mathcal{R}$ IS THE INTERIOR OF A POLYGON (MAYBE WITH BOUNDARY)

TRANSFORMATIONS (JUST A FUNCTION)

A TRANSFORMATION IS A RULE SO THAT FOR ANY POINT P OF THE PLANE WE ASSOCIATE A UNIQUE POINT Q.

WRITE $F(P) = Q$.

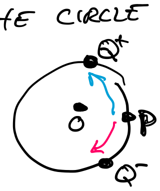
WE THINK OF F AS MOVING P TO Q

EXAMPLES
- IDENTITY TRANSFORMATION $I(P) = P$ $Id(P) = P$. INPUT = OUTPUT.
- CONSTANT TRANSF. PICK $X \in$ PLANE $F_X(P) = X$ NO MATTER WHAT P IS

ROTATIONS:
- LET O BE SOME POINT OF PLANE (THE CENTER OF ROTATION)
- LET Θ BE SOME NUMBER $-360 \leq \Theta \leq 360$

THE ROTATION ABOUT O BY $\Theta°$ IS THE TRANSFORMATION ρ_Θ SO THAT
- $\rho_\Theta(O) = O$
- IF $\Theta = 0$, $\rho_0(P) = P$ (SO ρ_0 IS I)
- IF $P \neq O$, LET C BE THE CIRCLE WITH CENTER O, RADIUS $|OP|$

LET Q BE ONE OF THE TWO POINTS Q^+ AND Q^- SO THAT



$\text{meas}(\widehat{PQ}) = |\Theta|$, IF $\Theta > 0$, WANT THE COUNTERCLOCKWISE

$\Theta < 0$ WANT CLOCKWISE

(WE CAN DEFINE CLOCKWISE BUT ITS LONG. READ IT) $F(P) = Q$

INJECTIVE, SURJECTIVE
BIJECTIVE

$F(P) = F(Q) \Rightarrow Q = P$
1-1

ONTO
FOR ANY Q THERE IS A P SO THAT $F(P) = Q$

BOTH
BIJECTIVE
1-1 ONTO