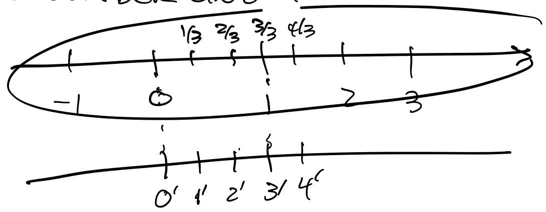
Parallels and betweenness

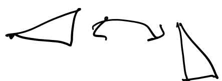
Wednesday, August 26, 2020 3:42 PM

ASSUME WE KNOW WHAT A NUMBER LINE IS.

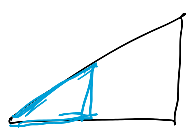


IN MIDDLE SCHOOL STUDENTS ARE TAUGHT

CONGRUENCE ⟺ "SAME SHAPE & SIZE"



SIMILARITY ⟺ SAME SHAPE NOT NECESS. SAME SIZE



VERY "FUZZY" IDEA.

FUNDAMENTAL OBJECTS IN GEOMETRY (DON'T REALLY DEFINE)

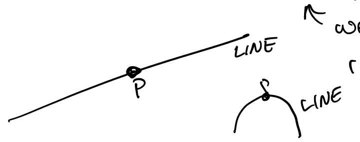
- POINT
- LINE
- PLANE

- A LINE CONSISTS OF POINTS
- PLANE CONTAINS LINES & THUS POINTS

- BASIC ASSUMPTIONS / POSTULATES / AXIOMS RELATE LINES/POINTS/PLANE
- DEFINITIONS.
- COMBINE TO MAKE PROVABLE STATEMENTS THEOREMS, LEMMAS PROPOSITIONS, COROLLARY ...

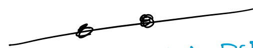
INFORMALLY, WE THINK OF A POINT AS A DIMENSIONLESS LOCATION & LINE IS A "STRAIGHT" LINE

← WHAT DOES THIS MEAN? DON'T CARE.



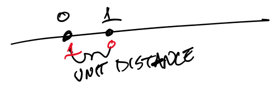
AND CONTAINS SOME POINTS (AT LEAST 2)

L1) THROUGH ANY TWO POINTS, THERE IS EXACTLY ONE LINE.



WE CAN PUT EVERY LINE IN 1-1 CORR. WITH A NUMBERLINE.

CALL ONE POINT 0, ANOTHER 1



⟹ EVERY LINE IS ARBITRARILY EXTENDABLE (INFINITE IN EACH DIRECTION)

(RULES OUT SPHERICAL GEOMETRY) & GET ∞-MANY POINTS.

DEF TWO LINES ARE DISTINCT IF AT LEAST ONE POINT BELONGS TO ONE AND NOT THE OTHER

LINES WHICH ARE NOT DISTINCT ARE THE SAME LINE

IF I HAVE FINITELY MANY LINES THERE IS SOME POINT IN PLANE NOT ON ANY OF THEM

DEF: TWO LINES ℓ_1 & ℓ_2 ARE PARALLEL IF THEY HAVE NO POINTS IN COMMON. WRITE $\ell_1 // \ell_2$.

LEMMA: LET ℓ_1, ℓ_2 BE DISTINCT LINES THEN EITHER $\ell_1 // \ell_2$ OR ℓ_1 HAS EXACTLY ONE POINT IN COMMON WITH ℓ_2

Pf/ IMMEDIATE FROM AXIOM (L1) ∴ IF ℓ_1 & ℓ_2 HAVE MORE THAN 1 PT IN COMMON, THEY HAVE 2, SO SAME LINE.

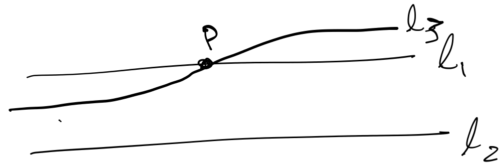
ARE THERE PARALLEL LINES?

(L2) PARALLEL POSTULATE
GIVEN A LINE ℓ AND A POINT P NOT ON THE LINE
THEN THERE IS AT MOST ONE LINE THROUGH P WHICH IS PARALLEL TO ℓ

OFTEN SAY "EXACTLY ONE"
IF YOU HAVE ENOUGH ROTATIONS, CAN PROVE THIS IS ENOUGH.

REALLY REALLY IMPORTANT ASSUMPTION
FROM IT FOLLOWS PYTHAGOREAN THM, RECTANGLES HAVE OPP. SIDES EQUAL.
NOTION OF SIMILARITY
SUM OF ANGLES IN $\triangle$ TRIANGLE IS $180°$

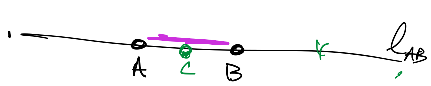
LEMMA: SPOZE ℓ_1, ℓ_2, ℓ_3 ARE 3 LINES
THEN IF $\ell_1 \parallel \ell_2$ AND $\ell_2 \parallel \ell_3$
THEN $\ell_1 = \ell_3$ OR $\ell_1 \parallel \ell_3$



SPOZE $\ell_1 \neq \ell_3$. IF $\ell_1 \not\parallel \ell_3$,
THEN THERE IS A P ON ℓ_1 AND ℓ_3.
VIOLATES PARALLEL POSTULATE
SO $\ell_1 = \ell_3$

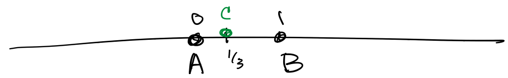
BETWEENNESS & SEGMENTS

SPOZE I HAVE A, B POINTS
$\Rightarrow$ LINE ℓ_{AB}



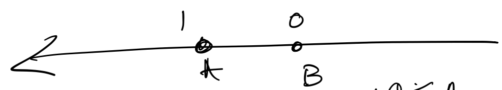
WANT TO DEFINE SEGMENT $\overline{AB}$
"ITS THE PART OF ℓ_{AB} BETWEEN A & B"

THE VICTORIA PLAN:
IDENTIFY ℓ_{AB} WITH A NUMBER LINE WITH $A=0$, $B=1$



$0 < 1/3 < 1$

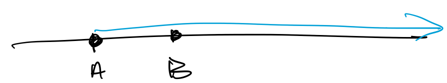
SO WE CAN SAY C IS BETWEEN A & B
IF WHEN WE MAKE A NUMBER LINE USING A & B, THEN
$A < C < B$.
WRITE $A * C * B$
WHAT IF I TOOK $B=0$ & $A=1$



$B < C < A$
CAN JUST SAY
$A * C * B$ OR $B * C * A$
ARE SAME.
KIND OF CLUNKY.

HOW CAN WE DEFINE RAY FROM A THRU B?
$\overrightarrow{AB}$ R_{AB}

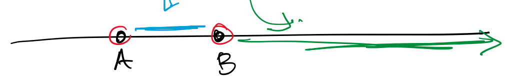
LINE CONTAINING A,B
$\overleftrightarrow{AB}$



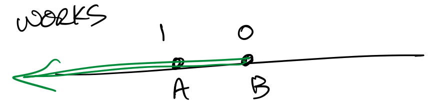
MAKE A NUMBER LINE WITH $A=0$, $B=1$ ANY + WILL DO

$\overrightarrow{AB} = \{ C \in \ell \mid \#(C) \geq 0 \}$
NUMBER ON MY # LINE.

OR CONSIDER ALL C WITH $A * C * B$
OR $A * B * C$
OR $C = A$ OR $C = B$



TO MAKE $\overrightarrow{BA}$ SAME WORKS



KATIE'S DEF:

$$\overline{AB} = \overrightarrow{AB} \cap \overrightarrow{BA}$$

NEXT TIME: POLYGONS
& SEPARATION.

NOW: SAAG PANEER & GOOD STUFF.