MAT515 Homework 8

Due Wednesday, November 11

1. **(a)** In $\triangle ABC$, prove that the angle bisector of $\angle A$ and the bisectors of the exterior angles at B and C is always concurrent. This point of concurrency is called an **excenter** of $\triangle ABC$; there are three such excenters of every triangle (one for each vertex).

   (b) Let X be one of the three excenters of $\triangle ABC$, and show that X is equidistant from the three lines $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, and $\overleftrightarrow{AC}$.

2. Let ℓ be a line passing through the centriod G of $\triangle ABC$ so that A is one half-plane with respect to ℓ and B and C lie in the other. Show that the distance from A to ℓ is equal to the sum of the distances from B to ℓ and from C to ℓ.

3. In triangle $\triangle ABC$, let D be the midpoint of $\overline{BC}$, and let M be the midpoint of the median $\overline{AD}$. Let E be the intersection of $\overrightarrow{AM}$ with $\overline{AC}$. Prove that $|EC| = 2|AE|$. (Hint: use Menelaus' Theorem, although you can also do it without it.)

4. Prove that in any triangle, the sum of the lengths of the medians is greater than $3/4$ of the perimeter of the triangle. (The **perimeter** of any polygon is the sum of the lengths of its edges.)