MAT515 Homework 7

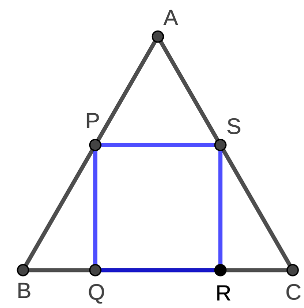
Due Wednesday, October 21

Problems marked with a * are optional/extra credit. Even if you don't do them, please at least think about them, then read and understand the solutions after they are posted.

1. In the figure at right, $\triangle ABC$ is equilateral, and $\square PQRS$ is a square. P lies on $\overline{AB}$, Q and R lie on $\overline{BC}$ and S lies on $\overline{AC}$.

If $|BC| = k$, what is the length of $\overline{QR}$?

Note: if you want to use a fact like "an equilateral triangle with a side of length 1 has altitude of length $\sqrt{3}/2$", you should prove that — preferably not just by the "method of lucky guess, then check."



2. Given $\triangle ABC$, let D be a point on $\overline{AB}$ and E be a point on $\overline{AC}$ so that $|AD|/|AB| = |AE|/|AC|$. Prove that $\overline{BE}$ and $\overline{CD}$ intersect on the median from A.

3. Let $\triangle ABC$ have angles of 30°, 60°, and 90°, with $\angle C$ being the right angle. Prove that the angle bisector of C, the median through C, the perpendicular bisector of $\overline{AB}$, and the altitude from C all lie on distinct lines.

4. **(a)** Let ℓ be a line and ρ be a rotation by θ degrees around a point O. Prove that if $\theta \neq 180$ and $\theta \neq -180$, then the lines $\rho(\ell)$ and ℓ intersect at some point Q, and each of the four angles at Q have measure either θ or $180 - \theta$ degrees.

(b) Let P_1 and P_2 be points on lines ℓ_1 and ℓ_2 respectively, and suppose ℓ_1 and ℓ_2 intersect at a point Q distict from P_1 and P_2.
Prove that there is a rotation ρ of θ degrees so that $\rho(\ell_1) = \ell_2$ and $\rho(P_1) = P_2$, where θ is the measure of one of the four angles at Q.

***5.** Let P be a point interior to triangle $\triangle ABC$ (not lying on one of the sides).
Prove that $|BP| + |PC| < |BA| + |AC|$.