MAT515 Homework 5

Due Wednesday, October 7

1. Given $\triangle ABC$, let F be the midpoint of $\overline{BC}$. Prove that $\overrightarrow{AF}$ is the angle bisector of $\angle A$ if and only if $|AB| = |AC|$.

2. **(a)** Prove that a dilation $\Delta_{O,r}$ of any convex set is convex.

(b) Prove that the dilation of polygon is a polygon.

(c) Prove that the dilation of a regular polygon is a regular polygon.

3. Let O be a point not on a circle $\mathcal{C}$ with center A and radius r, and let Δ be the dilation with center O and scale factor s. Prove that $\Delta(\mathcal{C})$ is a circle with center $\Delta(A)$ and radius sr.

4. Let P and Q be two distinct points in the plane, and let Δ_P and Δ_Q be the dilation with center P and scale factor r, and the dilation with center Q and scale factor s, respectively.

(a) If $s = 1/r$, show that $\Delta_P \circ \Delta_Q$ is a translation.

(b) If $rs \neq 1$, prove that there is a point X so that $\Delta_P \circ \Delta_Q$ is a dilation with center X.

5. Prove the converse of the alternate interior angle theorem (**G18**).

Theorem G19. *Let L be a transversal to a pair of distinct lines ℓ_1 and ℓ_2. If the alternate interior angles of L with respect to ℓ_1 and ℓ_2 are congruent, then ℓ_1 is parallel to ℓ_2.*
The same holds of the corresponding angles.