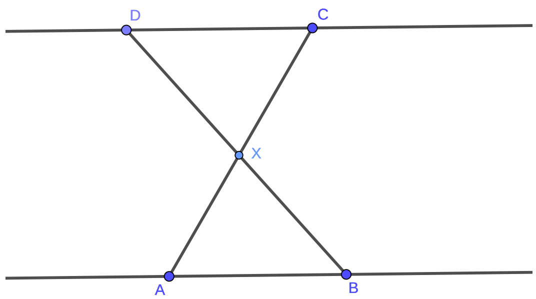
MAT515 Homework 3

Due Wednesday, September 23

1. Let ℓ be a numberline in the plane, with points A and B such that $\#(A) = 0$ and $\#(B) = 1$. Let $\rho_1 = \rho_{45,A}$ be the 45° (clockwise) rotation around A, and $\rho_2 = \rho_{90,B}$ be the 90° clockwise rotation about B.

As precisely as you can (without using cartesian coordinates), describe the two lines $\rho_1(\rho_2(\ell))$ and $\rho_2(\rho_1(\ell))$. Are the two lines equal?

2. Let lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ be parallel and such that segments $\overline{DB}$ and $\overline{AC}$ intersect at a point X with $\operatorname{dist}\left(X, \overleftrightarrow{AB}\right) = \operatorname{dist}\left(X, \overleftrightarrow{CD}\right)$. Prove that $|AB| = |CD|$.



3. The following problem shows that we *could* choose to forget about reflections and translations entirely, and just deal with reflections. But it is natural and convenient not to do so.

(a) Prove that every translation can be expressed as the composition of two reflections.

(b) Prove that every rotation can be expressed as the composition of two reflections.

This result is an *algebraic* afterthought regarding the behavior of basic isometries; in advanced mathematics, the algebraic viewpoint has paid enormous dividends. But keep in mind that it is merely an algebraic fact about how these transformations fit together, and it is, in fact, much more natural to *think* in terms of translations and rotations directly.

4. Prove that equidistant parallel lines intercept equal length segments on any transversal.

More specifically, let lines $\overleftrightarrow{AD}$, $\overleftrightarrow{BE}$, and $\overleftrightarrow{CF}$ be parallel to one another, and let $\overleftrightarrow{AC}$ and $\overleftrightarrow{DF}$ be transversals intersecting $\overleftrightarrow{BE}$ at B and E. Suppose $|DE| = |EF|$, and prove that $|AB| = |BC|$.

Hint: Draw the segments $\overline{EP}$ and $\overline{FQ}$ with P on $\overleftrightarrow{AD}$ and Q on $\overleftrightarrow{BE}$ and such that $\overleftrightarrow{AC} \parallel \overleftrightarrow{EP} \parallel \overleftrightarrow{FQ}$. Consider the translation T_{DE}, and prove that $T_{DE}(E) = F$ and then that $T_{DE}(P) = Q$.