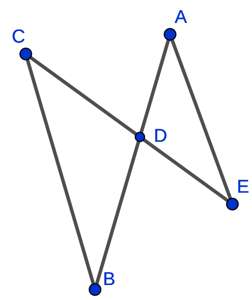
**MAT515 Homework 1**
Due Wednesday, September 9

**1**. Give all the reasons why the geometric figure with five vertices as shown below cannot be a polygon.




**2**. Let $A$ and $P$ be two distinct points on the line $\overleftrightarrow{AP}$, and let $\mathcal{R}$ be one of the two rays within $\overleftrightarrow{AP}$ issuing from $A$. Show that one and only one of the two rays issuing from $P$ either contains $\mathcal{R}$ or is contained in $\mathcal{R}$.

**3**. Let $\ell$ be a line in the plane, and let $A$, $B$, and $C$ be three points in the plane so that $A$ and $B$ are in the same half-plane with respect to $\ell$, and also $B$ and $C$ are in the same half-plane with respect to $\ell$. Prove that $A$ and $C$ are in the same half-plane with respect to $\ell$.

**4**. (a) Prove that any ray $\overrightarrow{AB}$ is convex.

(b) Prove that a closed half-plane is convex.

(c) Suppose that for each $i$, $\mathcal{C}_i$ is convex and also that $\mathcal{C}_i \subset \mathcal{C}_{i+1}$. There may be a finite number of sets $\mathcal{C}_i$, or there may be infinitely many of them. Prove that the union of all the sets $\mathcal{C}_i$ is also convex.

(d) Is it always true that the union of convex sets is convex? Provide a proof or give a counterexample.

**5**. Let $A$, $B$, and $C$ be three noncolinear points. Explain why the union of the three segments $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ must form a polygon; that is, each pair of segments can only intersect at one of the endpoints.

**6**. Let $\ell_1$ and $\ell_2$ be parallel lines, and let $\ell_3$ be a third line distinct from $\ell_1$. Prove that if $\ell_3$ intersects $\ell_1$ then it also must intersect $\ell_2$.

**7**. Imagine that the hands of a clock are rays emanating from the center of the clock. What is the measure of the angle between the hands when the time is 8:20?

**8**. A **triangular region** of the plane is defined to be the intersection of the three angles of a triangle.

(a) Show that any triangular region is convex.

(b) Let $\mathcal{T}$ be a triangular region in the plane. Show$^\dagger$ that if $P \in \mathcal{T}$ and $Q \notin \mathcal{T}$ then $\overline{PQ}$ must intersect one of the sides of $\mathcal{T}$ (perhaps at a vertex).

$^\dagger$without using Theorem 4.11, which we haven't discussed yet anyway