More stuff

$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos(A \pm B) = \cos A \cos B \pm \sin A \sin B$

Degree is a fine measure, but make calculus ugly
Arbitrary choice of units

5/2/2022

- Definitions
- State major theorems
- problems

Show that x^2 is continuous (#5 on last year's test)
f is continuous at c if
$\forall \varepsilon > 0 \quad \exists \delta$ s.t. $0 < |x-c| < \delta \Rightarrow |f(x) - f(c)| < \varepsilon$

Fix $\varepsilon > 0$. Need δ s.t. $|x-c| < \delta \Rightarrow |x^2 - c^2| < \varepsilon$

Note
$|x^2 - c^2| = |x-c||x+c| < \varepsilon$
$= |x-c| < \frac{\varepsilon}{|x+c|}$ ← this is my δ

take smallest x to make as big as possible

$$\frac{\varepsilon}{|3+c|} = \frac{\varepsilon}{3+c}$$

since $0 \le x \le 3$

Fix $\varepsilon > 0$. Let $\delta < \frac{\varepsilon}{c}$. Then if $|x-c| < \delta < \frac{\varepsilon}{c}$ then

$|x^2 - c^2| = |x+c||x-c|$
$< \quad \delta$

$|x-c| < \delta$
$\Rightarrow |x^2 - c^2| = |x+c||x-c| < \varepsilon$
$< |x+c|\delta < \varepsilon$
$\le |3+c| \left|\frac{\varepsilon}{c+3}\right|$
$= \varepsilon \left(\cancel{\frac{3+c}{c+3}}\right) \le \varepsilon$

★ can't have x in δ ★

Easy - x^2 is continuous on compact set $[0,3]$

bounded + closed

means closed bounded b/c has bounds

$\Rightarrow$ uniform cont on $[0,3]$
$\Rightarrow$ uniform cont on $(0,3)$

(not uniform compact for $x \ge 0$ aka if you make it unbounded)

$(0,3)$ not closed but fits in $[0,3]$

uniformly cont => δ doesn't depend on x

δ means deriv is bounded

$y=x$

$|x-c|<$

$|f(x)-f(c)|$

$|x-c| < \varepsilon$

$\delta = \varepsilon$

- Topology (open, closed, compact
- $\mathbb{R}$ is what?
- sequences + series
- Functions $f: A \to \mathbb{R}$
  - continuous, uniform cont
  - Differentiation → · Differentiable functions · integration
  - Integrable

Should be able to define a real #

(• The complete ordered field containing the rationals)

cauchy sequence converges to something

- You can represent it as an infinite decimal that may or may not repeat

$0.9\bar{9} = 1.0$

if $|x-y| < \varepsilon$
then for all $\varepsilon > 0$
then $x=y$

Prove $.9\bar{9} = 1.0$

$3\left(\frac{1}{3}\right) = (.\bar{3})3$

$$\begin{array}{r} 0.33 \\ 3\overline{)1.00} \\ -\,9 \\ \hline 10 \end{array}$$

$1 = .9\bar{9}$

$|1 - .9\bar{9}| < \varepsilon$ for any $\varepsilon > 0$

$\Rightarrow 1 = .\bar{9}$

$\{a_n\}$ $a_n \to L$

$\forall \varepsilon > 0 \quad \exists N_\varepsilon \in \mathbb{N}$ s.t. $\forall n > N_\varepsilon \quad |a_n - L| < \varepsilon$

i.e. a list of a_n which pile up on L, wait long enough all a_n are as close as you want

---

$f: A \to \mathbb{R}$ is differentiable at $c \in A$ if for all $\varepsilon > 0$, $\lim_{h \to 0} \frac{f(c+h) + f(h)}{h}$ exists.

$\Longleftrightarrow$

$f'(c) = L$ if

For every $\varepsilon > 0$, there is a δ so that $0 < |x - c| < \delta \Rightarrow \left| \frac{f(x) - f(c)}{x - c} \right| < \varepsilon$

Prove that $x^2 + x - 5$ is differentiable at π.

Easy - all polynomials are differentiable

or

$\frac{x^2 + x - 5}{x^2 + \sin x}$ $\Rightarrow$ is diff
$\Rightarrow$ is diff cos $\sin x$ is diff

$\pi^3 + \sin \pi \neq 0$, ratio