

# MATH 364

# Midterm

Halloween 2011

Name: \_\_\_\_\_ ID: \_\_\_\_\_

Question:	1	2	3	4	Total
Points:	20	20	20	20	80
Score:					

There are 4 problems on 4 pages in this exam (not counting the cover sheet). Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, extra papers, and discussions with friends are not permitted.** If you convinced Jack Milnor to do your exam for you, I will allow it. Otherwise, please do your own work.

You have 55 minutes to do this exam. (more than enough time for me to complete this exam, but maybe not enough for you. Life is unfair.)

20 pts.

1. Let  $X$  and  $Y$  be topological spaces, and let  $f : X \rightarrow Y$  be continuous and surjective. Show that if  $X$  is connected, then  $Y$  must be connected.

2. Let  $M$  be the image of  $\mathbb{R}^2$  under  $h : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  given by  $h(x, y) = (x^3, x^2, y)$ .

5 pts.

(a) Calculate  $Dh$ .

5 pts.

(b) What is the tangent space to  $M$  at  $(0, 0, 0)$ ?

5 pts.

(c) What is the tangent space to  $M$  at  $(1, 1, 1)$ ?

5 pts.

(d) Is  $M$  a smooth manifold? Justify your answer.

20 pts.

3. The set of all  $2 \times 2$  matrices with real entries and determinant 1 is called  $SL_2(\mathbb{R})$ . Show that  $SL_2(\mathbb{R})$  is a smooth 3-manifold by giving charts for it. Be sure to justify that your charts cover  $SL_2(\mathbb{R})$  and that they are smooth.

20 pts.

4. Consider the two disjoint, closed disks

$$D_1 = \{(x, y) \mid x^2 + y^2 \leq 1\} \quad \text{and} \quad D_2 = \{(x, y) \mid (x - 4)^2 + y^2 \leq 1\}.$$

Let  $M = D_1 \cup D_2$ .

Suppose  $f : M \rightarrow M$  a smooth function. Show that  $f$  must have a period 2 point; that is, there must be a point  $p \in M$  such that  $f(f(p)) = p$ .