Friday 10-21-11

NKC028

$f: I \to I$
an interval to an interval

$f: [0,1]$
smooth function
can draw a graph

$f: S^1 \to S^1$
the circle to the circle

How do we draw it?

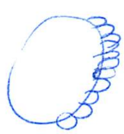
- graph of $f: A \to B$ can define it $\{(x, f(x))\ x \in A, f(x) \in B\}$;

let it = graph $(\Gamma(f))$

$\Gamma(f)$ is a subset of $A \times B$; $\Gamma(f) \subset A \times B$

$\Rightarrow S^1 \times S^1 = T^2$ (torus) we put a circle at every point

Ex.



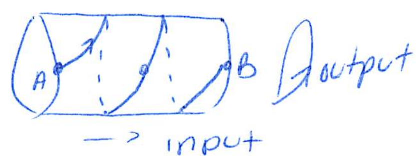
graph of $f: S^1 \to S^1$ is a torus knot.

Ex. $f(\theta) = 2\theta$

$f: S^1 \to S^1$ is continuous

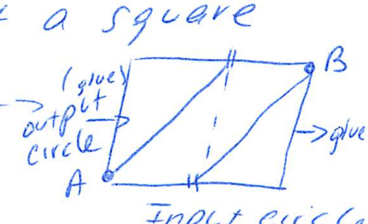
Ex.

we can see this easier if we cut open the torus

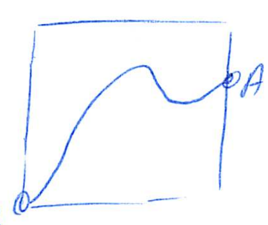


point A = point B = angle 0.

if we cut it again it will make it even easier (along the zero line) and unfold it. We get a square



Ex. 1



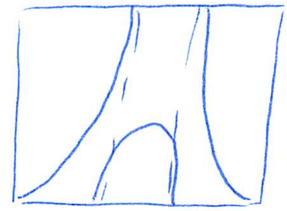
a graph of such f?
- No because it has to agree on the top & Bottom and Right and left A≠B

1

Ex 2.

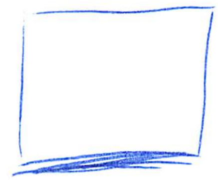
is a smooth $f: S^1 \to S^1$
the points agree

Ex 3.

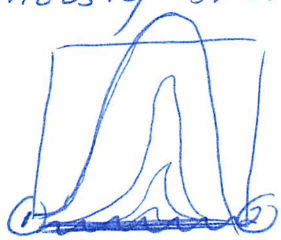


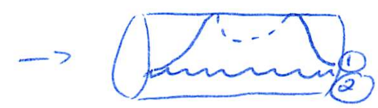
Is there something fundamentally deffetent from Ex 2 and Ex 3?

- Ex 2 goes around the hole once, and example 3 doesn't.
- We can continuly deform Ex 3 into

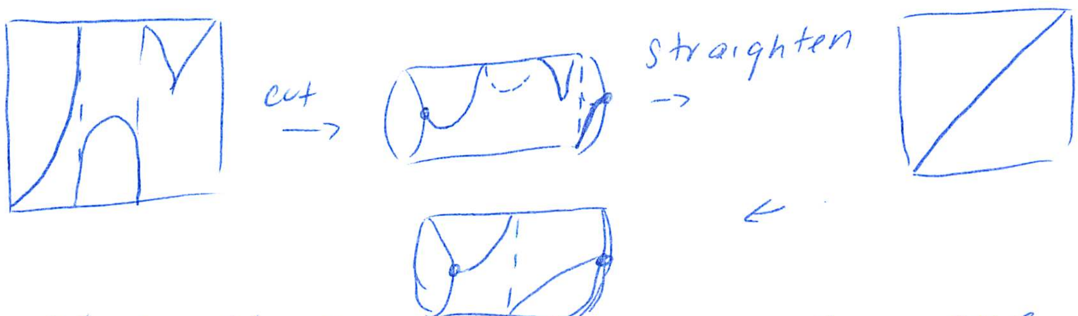


because we can continously shortening it in terms on torus





- In Ex 2.



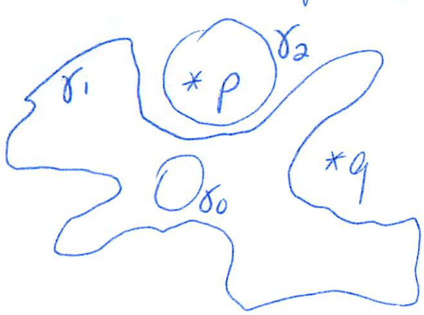
- we can see that there is something topological in the way they are fundamentally deffetent.

We are finding a Homotopy

- Two closed curves "γ" and "δ" are homotopic if we can continously deform γ into δ

Note: In the plane all closed curves are homotopic

Ex: $\mathbb{R}^2 \to \{p, q\}$



- γ_0 and γ_1 are homotopic because they dont contain p and q.
- γ_2 cannot be continously deformed into γ_0 and γ_1 $\therefore$ γ_2 is not homotopic.

- we want homotopy to be an equivalence relation

i.e. 1. $\gamma_1 \sim \gamma_2$ (homoptic) $\Rightarrow$ $\gamma_2 \sim \gamma_1$ (if γ_1 is homotopic to γ_2 then γ_2 is homotopic to γ_1.)

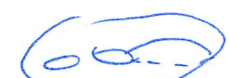
2. $\gamma_1 \sim \gamma_1$ for any γ we can continously deform itself onto itself.

3. $\gamma_1 \sim \gamma_2$ and $\gamma_2 \sim \gamma_3 \Rightarrow \gamma_1 \sim \gamma_3$

- Think about the sphere

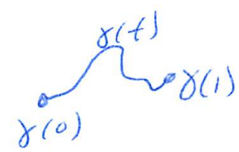
any closed loop on the sphere are homotopic to one another "all closed loops on S^n are homotopic."

- this is not so on the torus because there curves / loops can go through the hole.



Recall:

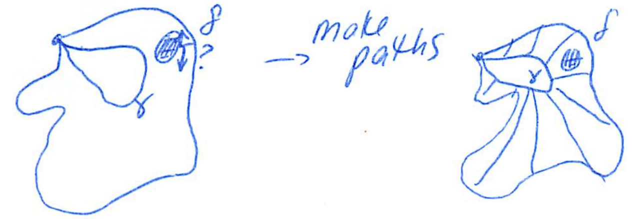
a path "γ" in X is continous function $\gamma(t)$ from $[0, 1]$ into X
start finish

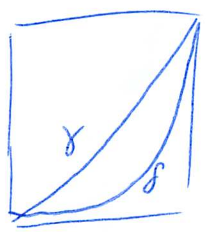


is a path with $\gamma(0) = \gamma 1$ (closed path/loop)

i.e a map $S^1 \to X$

Suppose $\gamma(t)$ and $\delta(t)$ for simplicity $\gamma(0)=\gamma(1)=\delta(0)=\delta(1)$ we want a whole family of paths for each t. so that $\delta(t) \to \gamma(t)$ and that the family of paths varies continously.



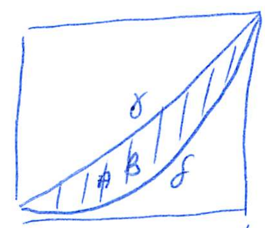


γ and δ are homotopic because we can put

$\gamma(t) = f(t,s)$ with $s=0$

$\delta(t) = f(t,s)$ with $s=1$

We can deform $\gamma(t)$ into $\delta(t)$ by letting "s" run from 0 to 1.



A and B cant differ too much.

want f to be continous in s as well as t.

want f to be the mapping of a square into the space instead of the curve.