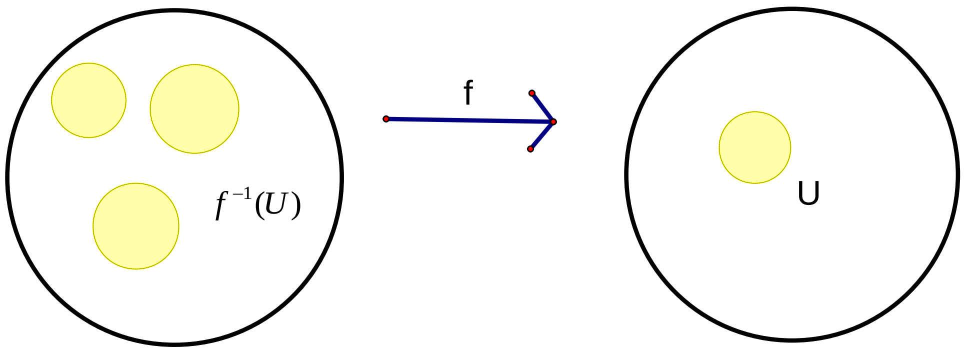
Notes MAT364 September 12th

Recall: $f: A \to B$ is <u>continuous</u> if for every open subset $U \subseteq B$, $f^{-1}(U)$ is open in A.



Proposition:

$f: X \to Y$ is continuous with $X \subseteq R^n$ and $Y \subseteq R^n \Leftrightarrow$ if x is a limit point of $B \subseteq X$, then $f(x)$ is a limit point of $f(B) \subseteq Y$

This just says:

Continuous functions send limit points to limit points.

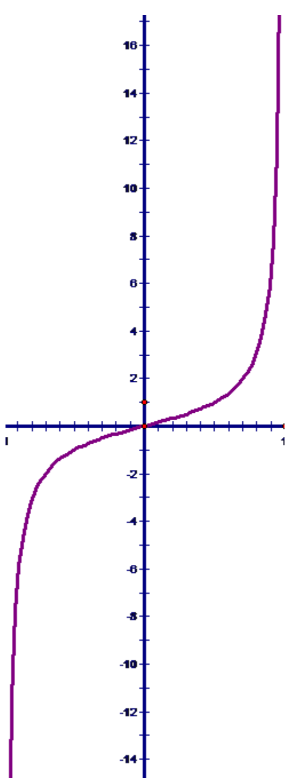
Goal of Topology:

Decide when two spaces are homeomorphic (the same in terms of topology).

Definition:

Two sets A and B are <u>homeomorphic</u>, if there is a continuous $f: A \to B$ with a continuous inverse $f^{-1}: B \to A$.

We can say f is a homeomorphism.



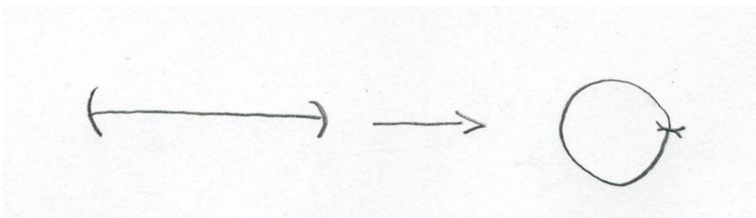
Examples:

$f: (-1,1) \to R$

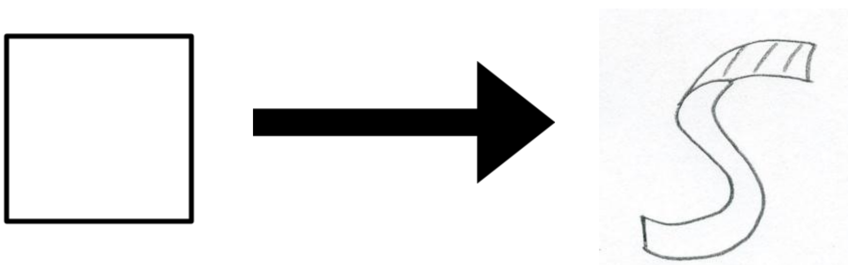
$f(x) = \tan(\frac{\pi}{2}x)$

This is a homeomorphism.

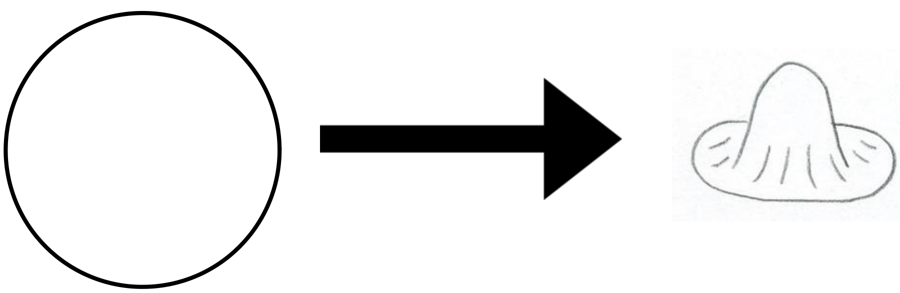
There is no homeomorphism from an open line to a circle that misses the origin because f^{-1} is not continuous.



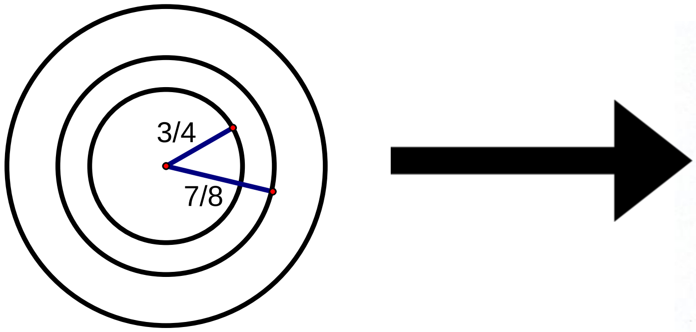
$f : unitsquare \to R^3$

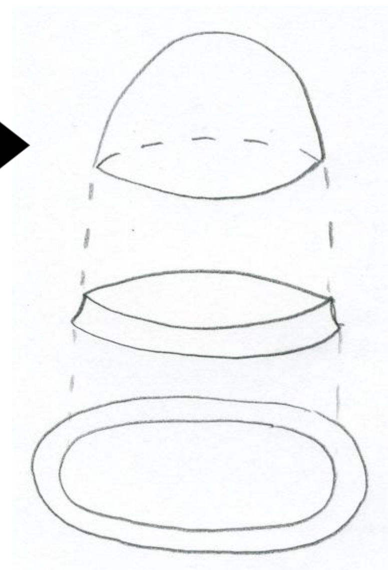


$g : unitcircle \to R^3$



To find a formula for $g(x,y)$ we need to define it in pieces and patch it together.





$$g(x,y)=\begin{cases}(x,y,\sqrt{1-x^2+y^2}) & x^2+y^2<\frac{3}{4}\\ (x,y,bendy(x,y)) & \frac{3}{4}\le x^2+y^2\le\frac{7}{8}\\ (x,y,0) & x^2+y^2>\frac{7}{8}\end{cases}$$

Bendy(x,y) has to be 0 if $x^2+y^2=\frac{7}{8}$ and $\frac{1}{2}$ if $x^2+y^2=\frac{3}{4}$

If we define the function g in this piecewise manner, g is a homeomorphism.

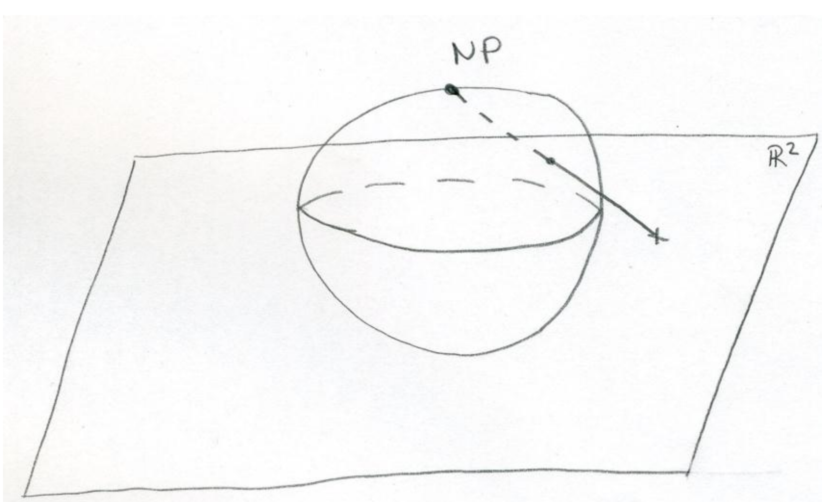
We cannot do this with a sphere because it is not homeomorphic to the plane.

We can use Stereographic Projection:

There is a homeomorphism f from $S^2-\{north\ pole\}$ to R^2

$S^2=\{x^2+y^2+z^2=1\}$

NP=(0,0,1)



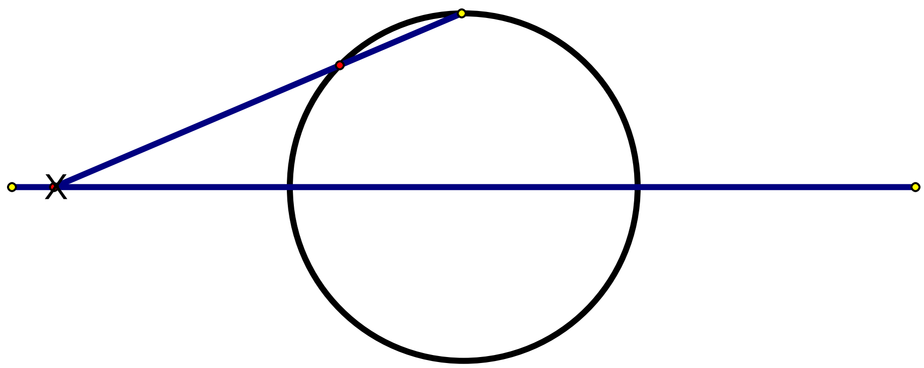
$f(x,y,z) =$ the point where the line between the north pole and the point (x,y,z) hits R^2

We can see that f(southern disk)=unit disk

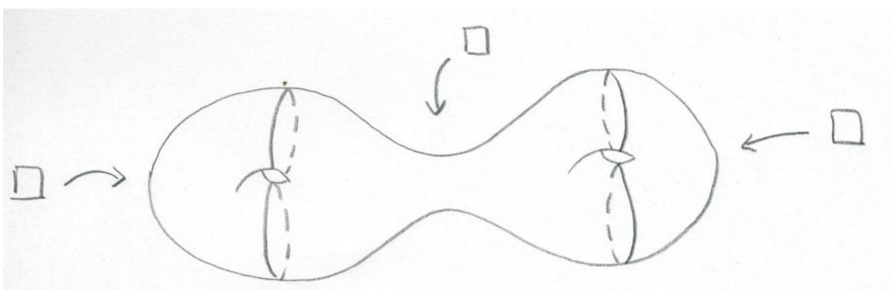
f(northern pole) is outside (not in R^2)

The same works for a line and the unit disk:

There is a homeomorphism from $R \cup \{\infty\}$ to the unit disk.



$S^3 = \{(x,y,z,w) \mid x^2 + y^2 + z^2 + w^2 = 1\} \approx R^3 \cup \{point\}$



To describe M, we give a collection of parameterizations (charts), each has to be a homeomorphism $f_i : U \subset R^n \to M$.

Definition: A <u>manifold</u> $M \subseteq R^m$ is a set M and a collection of charts $f_i : U \to M \subseteq R^m$ and $U \subseteq R^n$, so that $\bigcup_i f(U) = M$

We need continuity along the edges and the $f_i s$ have to be homeomorphisms.