MAT364, Homework 3
due wednesday 9/28

1. Show that $A = \{(x, y) \mid 1 < x^2 + y^2 < 2\}$ and $B = \mathbb{R}^2 \smallsetminus \{(0, 0)\}$ are diffeomorphic.

   More specifically, construct an explicit map $f : B \to A$ and show that

   - f is a continuous bijection
   - Df is nonsingular everywhere (calculate Df and show that the determininant is nonzero for all $(x, y) \in B$).
   - Df^{-1} is nonsingular (explicitly find $f^{-1} : A \to B$ and calculate Df^{-1}, then show the determinant is nonzero).

2. Give an explicit example (that is, write down a parameterization) of a function $g : \mathbb{R}^2 \to \mathbb{R}^3$ for which

   - g is injective
   - $Dg(0, 0)$ has rank 0
   - the rank of $Dg(x, y)$ is 2 at all $(x, y) \neq (0, 0)$.

3. Let M be the image of $\mathbb{R}^2$ under the map $h : \mathbb{R}^2 \to \mathbb{R}^3$ given by $h(x, y) = (x^3, x^2, y)$.

   (a) Does $h^{-1} : M \to \mathbb{R}^2$ exist?

   (b) What is the tangent space to M at $(0, 0, 0)$?

   (c) What is the tangent space to M at $(1, 1, 1)$?

   (d) Is M a smooth manifold?