MAT364, Homework 1
due wednesday 9/14

1. Recall that for a set A in $\mathbb{R}^n$, its closure is the union of its interior points and those in its frontier, that is
$$\mathrm{Cl}(A) = \mathrm{Int}(A) \cup \mathrm{Fr}(A).$$
Prove for any such A, $\mathrm{Cl}(A)$ is a closed set.

2. Suppose x and y are points in $\mathbb{R}^n$. The closed line segment between x and y is defined as
$$[x, y] = \{tx + (1 - t)y \mid 0 \leq t \leq 1\}.$$
A subset $A \subset \mathbb{R}^n$ is called convex if for every x and y in A, every point of the segment $[x, y]$ is also contained in A.

   Prove that if x_0 is any point in $\mathbb{R}^n$, the open disk $D_r(x_0)$ and the closed ball $B_r(x_0)$ are both convex sets.

3. Give an example of a countably infinite family of closed sets $A_1, A_2, A_3, \ldots$ such that $\bigcup_{i=1}^{\infty} A_i$ is not closed.

4. True or False: "If A and $A \cup B$ are open, then B must be open. If true, give a proof. If false, give a counterexample.

5. Consider the real line $\mathbb{R}$ as the x-axis in $\mathbb{R}^2$. If B is a closed subset of $\mathbb{R}$, prove that it is also closed when viewed as a subset of $\mathbb{R}^2$.

   Is the same property true for open sets? That is, if A is an open subset of $\mathbb{R}$, is A also open when viewed as a subset of $\mathbb{R}^2$? Prove or give a counterexample.