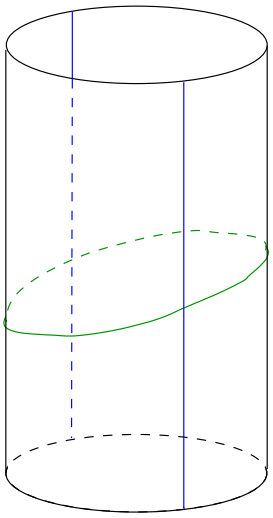
MAT360

Here is a problem I thought about for the final, but decided to use something else instead. It is worth you thinking about.

1. Consider the following interpretation of the terms "point", "line", and "incidence":

   - A point is any point on a fixed cylinder of radius 1 in $\mathbb{R}^3$, that is, a triple of real numbers (x,y,z) such that $x^2+y^2=1$ and z is any real number.
   - A line is the intersection of this cylinder with any plane in $\mathbb{R}^3$. Thus, lines are either a pair of vertical lines (which we get by intersecting with planes of the form $ax+by=c$) on the cylinder, or a circle going around the cylinder (which are obtained by intersecting with planes of the form $ax+by+z=d$.) A line of each type is shown at right.
   - Incidence is the usual relation.

   (a) Does this define an incidence geometry? That is, do axioms **I1**, **I2**, and **I3** hold? Fully justify your answer.

   (b) Does this model define a Euclidian, elliptic, or hyperbolic geometry (or none of the those)? Again, fully justify your answer.

   (c) Suppose we change the model so that a line can only be a Euclidean circle and not a pair of vertical lines (that is, we don't intersect with vertical planes, only "tilted" ones). Does this change your answer to the previous parts? How?