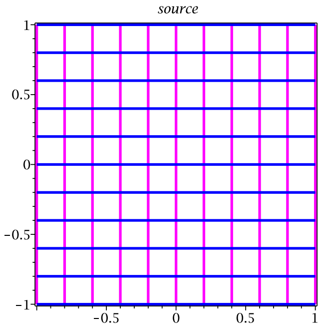
Our goal is to try to understand the behaviour of $f(z) = z^2$ graphically.

First try: Let's take a rectangular grid in $\mathbb{C}$ and see what f does to it.

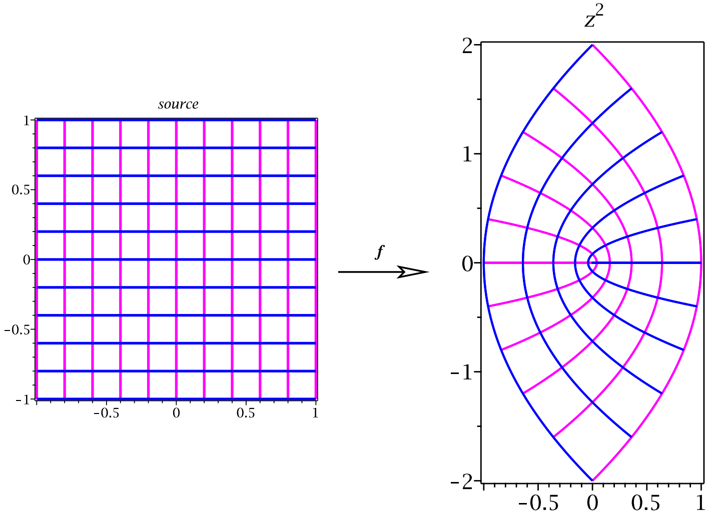
Does this convey everything? How do the number of lines compare between right and left?

Our goal is to try to understand the behaviour of $f(z) = z^2$ graphically.
First try: Let's take a rectangular grid in $\mathbb{C}$ and see what f does to it.



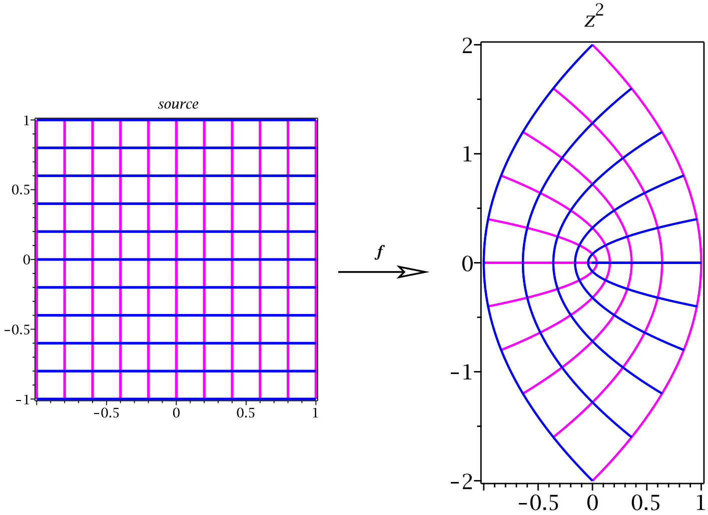
Does this convey everything? How do the number of lines compare between right and left?

Our goal is to try to understand the behaviour of $f(z) = z^2$ graphically.
First try: Let's take a rectangular grid in $\mathbb{C}$ and see what f does to it.



Does this convey everything? How do the number of lines compare between right and left?

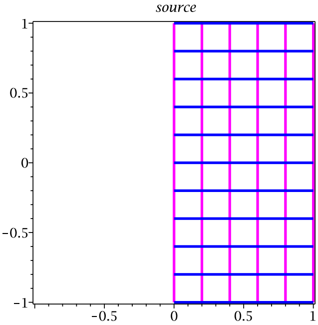
Our goal is to try to understand the behaviour of $f(z) = z^2$ graphically.
First try: Let's take a rectangular grid in $\mathbb{C}$ and see what f does to it.



Does this convey everything? How do the number of lines compare between right and left?

Let's try again, looking just at the image of the right half-plane $(\mathrm{Re}(z) > 0)$ first.

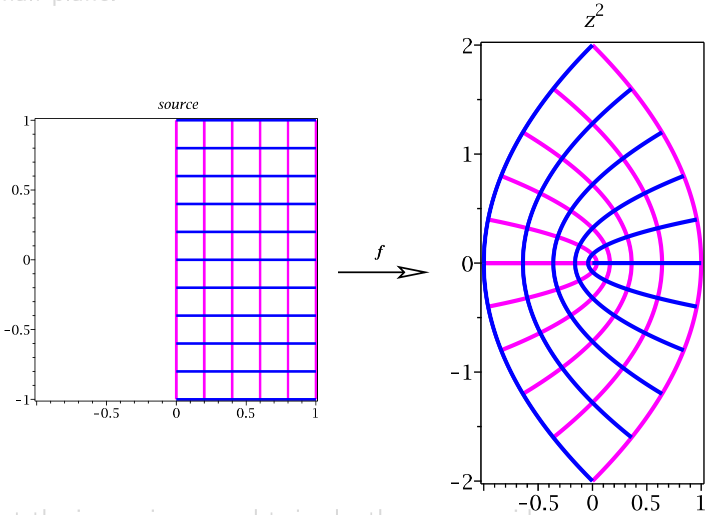
Now add in the left half-plane.



It should be clear that the image is covered twice by the source grid.

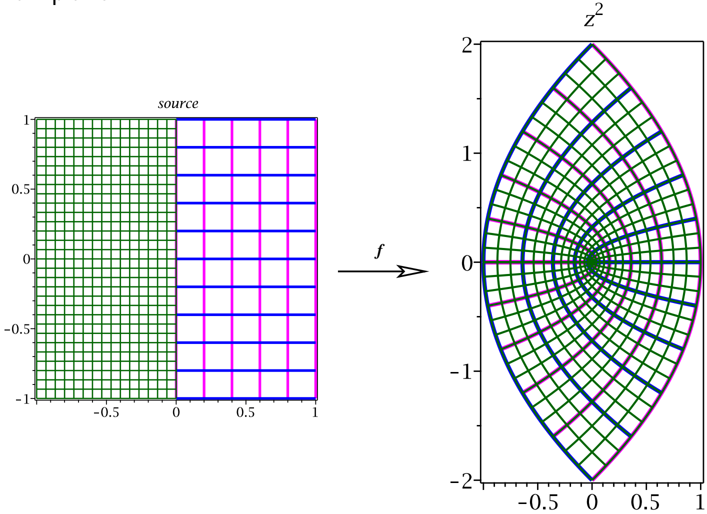
Let's try again, looking just at the image of the right half-plane $(\mathrm{Re}(z) > 0)$ first.

Now add in the left half-plane.



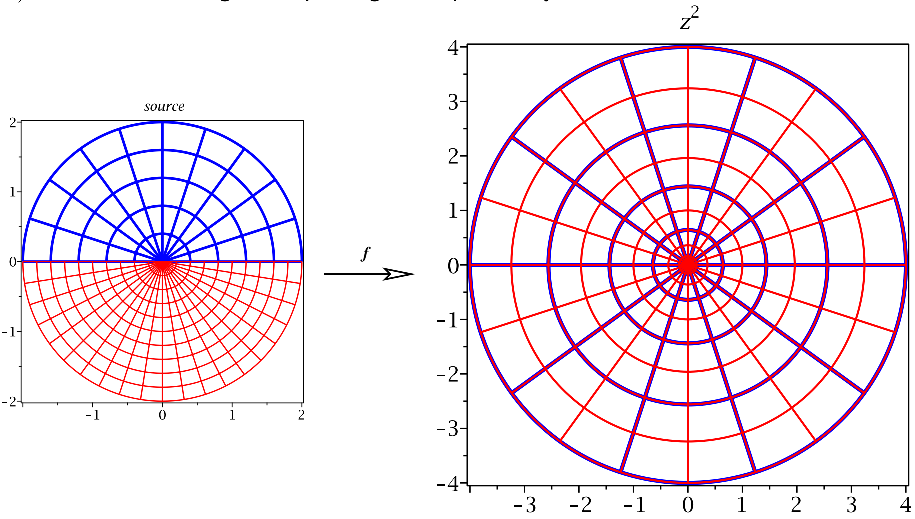
It should be clear that the image is covered twice by the source grid.

Let's try again, looking just at the image of the right half-plane ($\mathrm{Re}(z) > 0$) first.
Now add in the left half-plane.



It should be clear that the image is covered twice by the source grid.

Since $f(re^{i\theta}) = re^{2i\theta}$, the image of a polar grid is quite easy to understand.



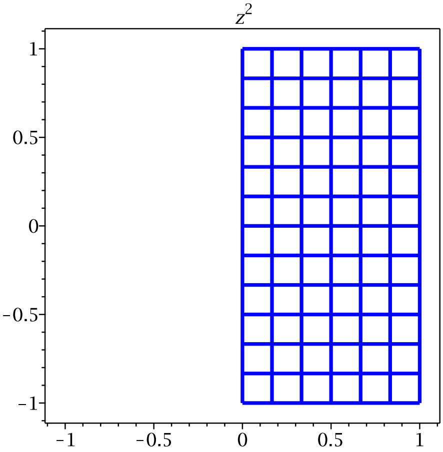
Here we color $0 \leqslant \theta \leqslant \pi$ in blue and $-\pi \leqslant \theta \leqslant 0$ in red.

Here's a slightly different approach. Start with a grid in the image plane, and see if we can find its *preimage*. This is done in detail in the textbook (§2.14).

Since $f(x+iy) = (x^2 - y^2) + (2xy)i = u + iv$, it can be shown vertical lines of the form $u = c$ $(c > 0)$ are the image of branches of hyperbolæ $x^2 - y^2 = c$, and horizontal lines $v = k$ are the image of branches of hyperbolæ $xy = k/2$.

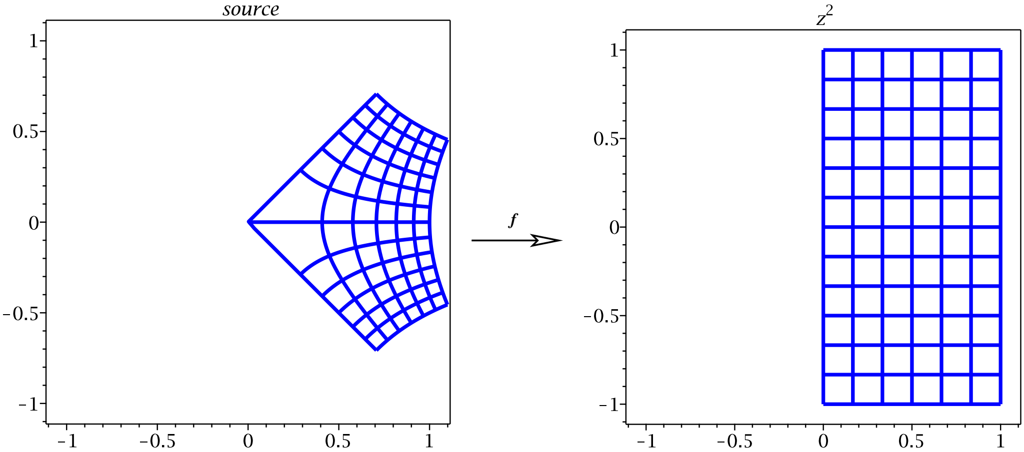
Here's a slightly different approach. Start with a grid in the image plane, and see if we can find its *preimage*. This is done in detail in the textbook (§2.14).

Since $f(x+iy) = (x^2 - y^2) + (2xy)i = u + iv$, it can be shown vertical lines of the form $u = c$ $(c > 0)$ are the image of branches of hyperbolæ $x^2 - y^2 = c$, and horizontal lines $v = k$ are the image of branches of hyperbolæ $xy = k/2$.



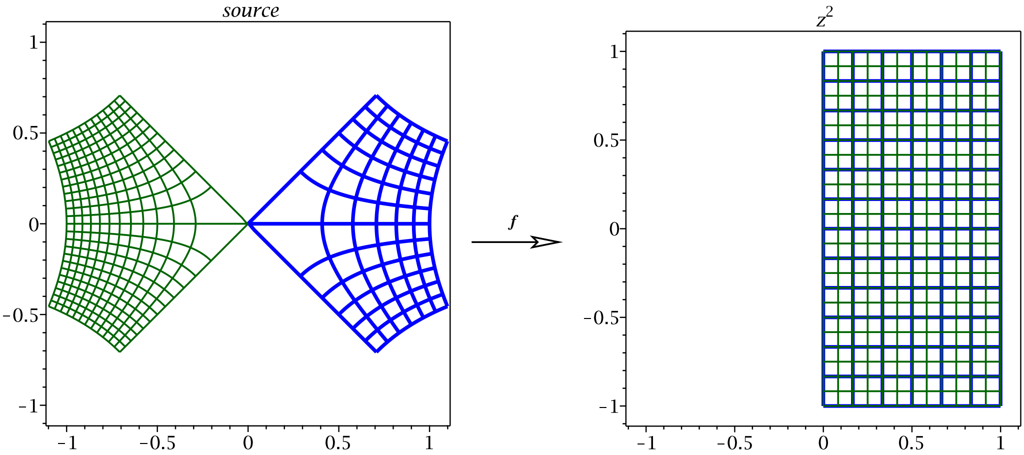
Here's a slightly different approach. Start with a grid in the image plane, and see if we can find its *preimage*. This is done in detail in the textbook (§2.14).

Since $f(x+iy) = (x^2 - y^2) + (2xy)i = u + iv$, it can be shown vertical lines of the form $u = c$ $(c > 0)$ are the image of branches of hyperbolæ $x^2 - y^2 = c$, and horizontal lines $v = k$ are the image of branches of hyperbolæ $xy = k/2$.



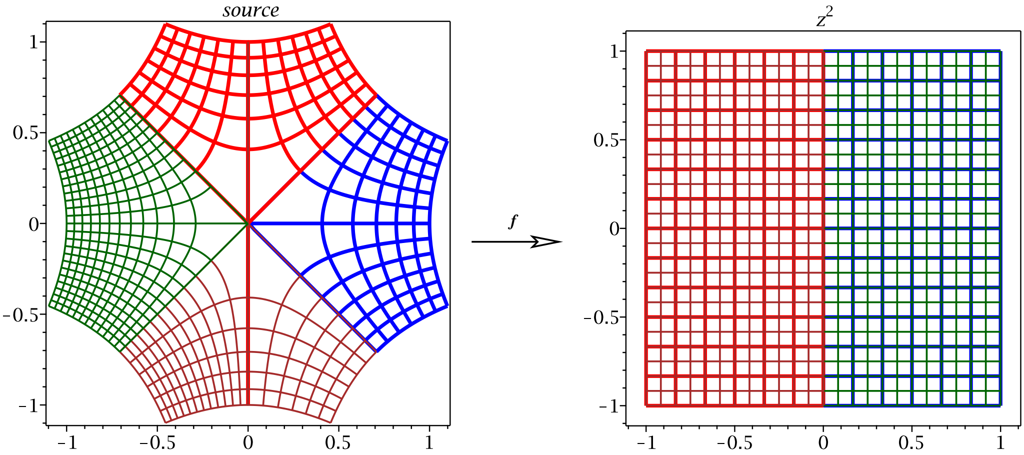
Here's a slightly different approach. Start with a grid in the image plane, and see if we can find its *preimage*. This is done in detail in the textbook (§2.14).

Since $f(x+iy) = (x^2 - y^2) + (2xy)i = u + iv$, it can be shown vertical lines of the form $u = c$ ($c > 0$) are the image of branches of hyperbolæ $x^2 - y^2 = c$, and horizontal lines $v = k$ are the image of branches of hyperbolæ $xy = k/2$.



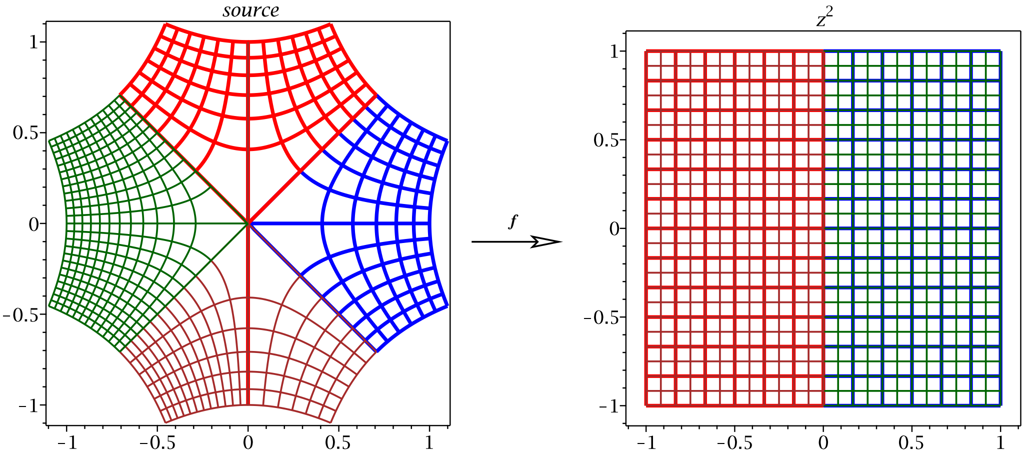
Here's a slightly different approach. Start with a grid in the image plane, and see if we can find its *preimage*. This is done in detail in the textbook (§2.14).

Since $f(x+iy) = (x^2 - y^2) + (2xy)i = u + iv$, it can be shown vertical lines of the form $u = c$ $(c > 0)$ are the image of branches of hyperbolæ $x^2 - y^2 = c$, and horizontal lines $v = k$ are the image of branches of hyperbolæ $xy = k/2$.



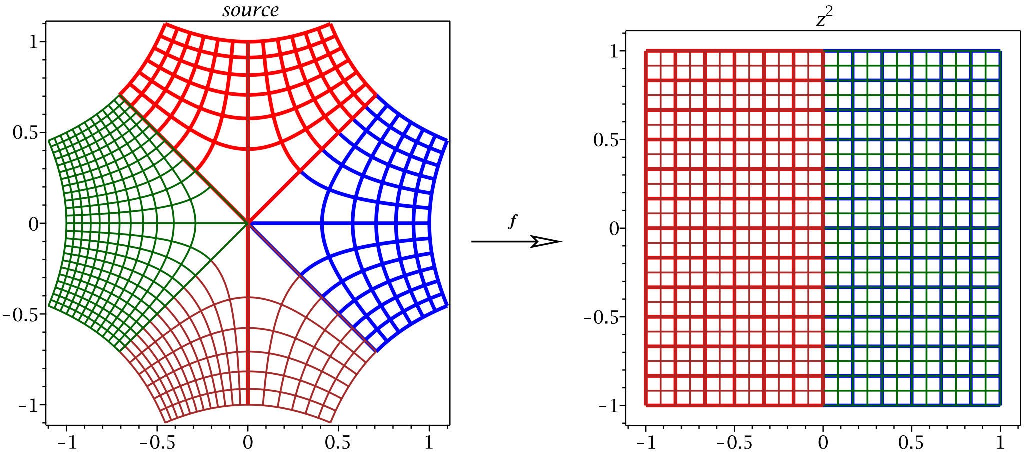
Here's a slightly different approach. Start with a grid in the image plane, and see if we can find its *preimage*. This is done in detail in the textbook (§2.14).

Since $f(x+iy) = (x^2 - y^2) + (2xy)i = u + iv$, it can be shown vertical lines of the form $u = c$ $(c > 0)$ are the image of branches of hyperbolæ $x^2 - y^2 = c$, and horizontal lines $v = k$ are the image of branches of hyperbolæ $xy = k/2$.

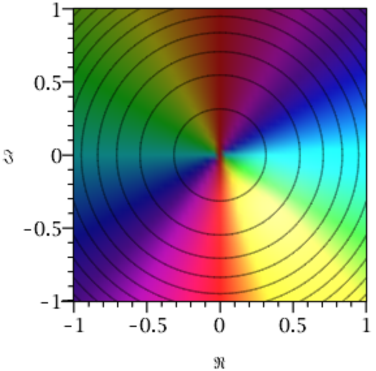


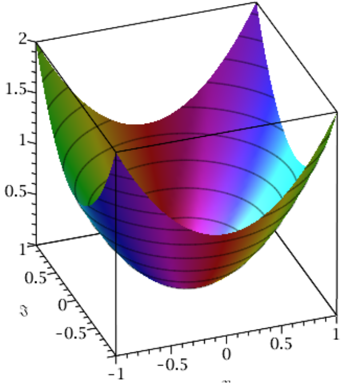
Here's a slightly different approach. Start with a grid in the image plane, and see if we can find its *preimage*. This is done in detail in the textbook (§2.14).

Since $f(x+iy) = (x^2 - y^2) + (2xy)i = u + iv$, it can be shown vertical lines of the form $u = c$ $(c > 0)$ are the image of branches of hyperbolæ $x^2 - y^2 = c$, and horizontal lines $v = k$ are the image of branches of hyperbolæ $xy = k/2$.



Yet another approach: Draw the graph of $|f(z)|$ as a surface in $\mathbb{R}^3$, but also color it to indicate the argument of $f(z)$. This is a little confusing, since if there is a place on the surface with the same color at the same height, these represent the same image point.

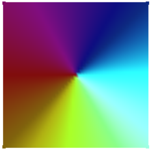


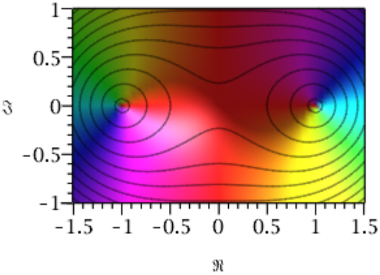


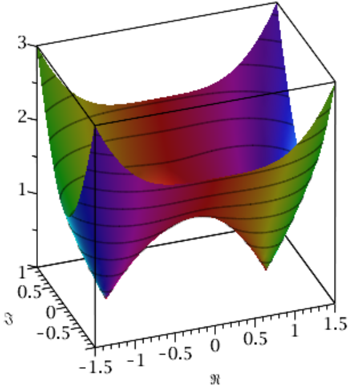
See also the Complex Function Viewer at `http://davidbau.com/conformal/`

For variety, here is the same with $f(z) = z^2 - 1$.

Yet another approach: Draw the graph of $|f(z)|$ as a surface in $\mathbb{R}^3$, but also color it to indicate the argument of $f(z)$. This is a little confusing, since if there is a place on the surface with the same color at the same height, these represent the same image point.







See also the Complex Function Viewer at `http://davidbau.com/conformal/`
For variety, here is the same with $f(z) = z^2 - 1$.