

MAT 342

Midterm

March 13, 2019

Name: _____ ID: _____

Question:	1	2	3	4	5	Total
Points:	10	10	10	10	10	50
Score:						

There are 5 problems on 5 pages (plus this cover sheet) in this exam. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, calculators, extra papers, and discussions with friends are not permitted. No electronic devices may be used AT ALL.** If you brought a telepathic duck with you to the exam, feel free to use it to read my mind about the correct answers on the questions. Other extra-sensorially enhanced beings are not permitted however, and their use will constitute academic dishonesty.

Leave all answers in exact form (that is, do *not* approximate π , square roots, and so on.)

You have 52 minutes to complete this exam.

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10 pts

1. Simplify each of the following, writing your answer in the form $a + ib$ where a and b are real numbers (or sets of real numbers). Justify your answers fully.

(a) $\frac{1 - i}{1 + \sqrt{3}i}$

(b) $\left(\frac{1 - i}{\sqrt{2}}\right)^i$

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10 pts

2. For each of the functions below, state all values of z for which the function is analytic. (If there are none, say so). Justify your answer fully.

(a) $f(z) = \frac{\bar{z}}{|z|^2}$

(b) $g(z) = \bar{z}^2$

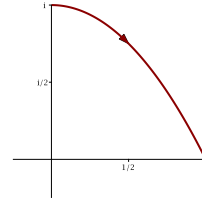
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10 pts

3. Let $\gamma(t) = t + (1 - t^2)i$, with $0 \leq t \leq 1$. Calculate the integral

$$\int_{\gamma} z^2 + \frac{1}{z} dz .$$



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4. Consider the domain \mathcal{D} consisting of the complex plane with the positive imaginary axis removed (0 is also not included in \mathcal{D}).

Let f be the multivalued function $z \mapsto z^{1/4}$. *Explicitly* describe the branch of f for which $f(1) = -i$, and calculate $f(-1)$.

$f(-1) =$ _____

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5. Let $E = \{ z \in \mathbb{C} \mid \operatorname{Im} z > 0 \text{ and } \operatorname{Re} z \geq 0 \}$ and describe the set $f(E)$ where $f(z) = -z^3$. As part of your answer, include a sketch of $f(E)$, using solid lines to indicate boundaries which are included and dashed lines to indicate boundaries not included. Shade the interior of $f(E)$ (if there is one).