Stony Brook University
Mathematics Department
Oleg Viro

Applied Complex Analysis
MAT 342
April 15, 2008

Final Exam

Examination time: 8:00-10:30 am. No electronic devices, books or notes. Show all your work.

Name _____

Student ID # _____

Problem #	Points/total
1	/5
2	/5
3	/10
4	/10
5	/20
6	/15
7	/15
8	/20
Total	/100

Name _____

Problem 1 (5pt). Find all complex values of $1^{\sqrt{2}}$. Give the answer in the form $a + ib$.

Name _____

Problem 2 (5pt). Let $f(x+iy) = u(x,y) + iv(x,y)$ be an analytic function. Prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

Name _____

Problem 3 (10pt). Find the image of the set $\{(x, y) \in \mathbb{C} \,|\, 0 < x < 1\}$ under the transformation

$$f(z) = \frac{z-1}{z-2}.$$

Name _____

Problem 4 (10pt). Evaluate the integral

$$\int_C |z|\,\bar{z}\,dz,$$

where C is the boundary of the set $\{z \in \mathbb{C}\,|\,|z| \leq 1,\ \mathrm{Im}\,z \geq 0\}$ taken in the counterclockwise direction.

Name _____

Problem 5 (20pt). In which domains is the function $f(z) = \dfrac{1}{(z-1)(z-i)}$ represented by Laurent series in powers of z? Find the Laurent series in the unbounded domain.

Name _____

Problem 6 (15pt). Evaluate the integral

$$\oint_C \frac{z+1}{e^z+1}\,dz,$$

where C is the circle $|z| = 4$ taken in the counterclockwise direction.

Name _____

Problem 7 (15pt). Find and classify all singularities of the function $f(z) = \dfrac{z-\pi}{\sin 2z}$. Find the principal part of Laurent expansion of f about $z = 2\pi$.

Name _____

Problem 8 (20pt). Evaluate the integral

$$\int_0^\infty \frac{x^2}{x^4+1}\,dx.$$

Explain carefully each step in your calculation.