MAT342 Homework 8

Due Wednesday, April 10

1. (a) Let f be continuous on a closed, bounded region R, and analytic on the interior of R. If f is not constant and nonzero throughout R, prove that $|f(z)|$ has a *minimum* value which occurs on the boundary of R and never in the interior.
Hint: apply the maximum principle to $g(z) = 1/f(z)$.

(b) Show that the condition $f(z) \neq 0$ is necessary in the previous part. That is, give an example of a nonconstant, analytic function $f(z)$ and a closed bounded region R where $|f(z)|$ has a minimum interior to R (which is smaller than at any point on the boundary).
Hint: the identity function $f(z) = z$ will work.

2. Let R be the rectangular region $0 \leq \operatorname{Re} z \leq 1$, and $0 \leq \operatorname{Im} z \leq \pi$. Determine the values z where the norm of the function e^z attains its maximum and minimum values.

3. Using the definition of the limit of a sequence, show that $\displaystyle\lim_{n\to\infty}\left(2i + \frac{i^n}{n^2}\right) = 2i$.

4. Obtain the Maclaurin series $\displaystyle z\cosh(z^2) = \sum_{n=0}^{\infty} \frac{z^{4n+1}}{(2n)!}$. For what z does it converge?

5. Using the fact that $e^z = e \cdot e^{z-1}$, obtain the Taylor series $\displaystyle e^z = e \cdot \sum_{n=0}^{\infty} \frac{(z-1)^n}{n!}$.

6. Let $\displaystyle f(z) = \frac{-1}{(z-1)(z-2)} = \frac{1}{z-1} - \frac{1}{z-2}$. Write the Laurent series for $f(z)$ when $1 < |z| < 2$.
Hint: rewrite the first term of f in terms of $\frac{1}{1-1/z}$ and the second using $\frac{1}{1-(z/2)}$, then use geometric series.