MAT 331, Spring 2024

Project 1: Least-Squares Fitting

Due Saturday, March 16

In this project, you are to fit a line to some data in three different ways. First, by using the "usual" least squares method, which minimizes the sums of the squares of the vertical distances between the data and the line. Then, find the line given by assuming that the x variable may contain the errors and the y-values are exact (and hence minimize the sums of squares of the *horizontal* distances). Finally, assume that both x- and y- values are approximate, and find the line that minimizes the sum of the squares of the *absolute* distances from the points to the line. See the three figures below.



In the third case (minimizing the absolute Euclidean distance), the equations you need to solve to find the minimum will be nonlinear, and you will discover that there are two critical points. Only one is the minimum; what line does the other critical point correspond to? Why *must* it be there? (You may also get a line with a huge slope and intercept— this is due to rounding errors and may be thought of as a critical point "at infinity"). You might find it helpful to use plot3d to draw a picture of the surface that you are finding the minimum of.

You should, of course, also plot the 3 lines you find along with the data points on the same graph, including appropriate labeling.

Your project should be written as a paper which explains clearly each step you take and why. In particular, you *must* give a careful derivation of the functions you minimize— you cannot merely quote the formula for the distance from a point to a line. You may, of course, assume that the formula for the distance between two points in the plane is known to the reader. Your paper should be written so that someone who has a good working knowledge of mathematics but has never taken this class will understand what you are doing and why.

Your explanation should focus on the actual mathematical ideas, not on the syntax of various Maple commands. For example, rather than saying something like "the Maple command diff(f(x,y),x) calculates the derivative $\frac{\partial f}{\partial x}$ ", instead explain why you are computing the partial derivative, and then do it. Focus on the idea and the explanation, not the syntax of the computer language.

You should use Maple to do the actual computations. While it would make for a more readable paper to intersperse your Maple calculations along with your exposition, you may write your exposition separately and append the Maple worksheet. However, *do not* include false starts, mistakes, or irrelevant calculations in your finished product. Pay attention to spelling, grammar, and sentence structure. **The expository part of this project counts as much as the actual calculations, and you should give it a lot of attention.**

There is a file containing a Maple command to assign the data points to a list called data at

https://www.math.stonybrook.edu/ scott/mat331.spr24/problems/proj1data/NETID.txt

where NETID is your Stony Brook NetID. Use either the read or HTTP[Get] command to load this data (or the ExecuteFromWeb command from the class page) as described here, or just paste the data into the worksheet.

In order to help you confirm your work, there is also a data file called testcase.txt which should yield (rounded to 4 places):

Vertical: y = -0.3009x + 4.7784Euclidean: y = -0.3142x + 4.7944Horizontal: y = -0.4541x + 4.9633