9. (*expires 2/23*) Given the set of points

$$(0,3), (1,0), (2,4), (3,4), (4,-1), (6,4), (7,0), (9,2), (10,10),$$

Find the polynomial of degree 8 that passes through all of them. If you wish, you can use `CurveFitting[PolynomialInterpolation]`, or you can calculate all of the relevant equations.

Then find the polynomial of degree 9 which passes through these points and also has a critical point at $x = 6$.

Also find the polynomial of degree 10 passing through the points with critical points at both $x = 1$ and $x = 6$.

Finally, make a plot displaying all three graphs, together with the data points. Be sure that your plot shows the data points clearly (as points, not connected lines), and clearly distinguishes the three functions. Including a legend (see `plot,options`) is one good way to do this.

To avoid typing in the points, you can load them from the web at `https://www.math.stonybrook.edu/~scott/mat331.spr24/problems/extras/polydata.txt`, which defines a list called `polydata` containing them.

10. (*expires 2/23*) Using the same data as the previous problem, find both the "natural" cubic spline which interpolates the data, and also the "periodic" cubic spline.

Also calculate the cubic spline which has derivative 0 at $x = 0$ and $x = 10$, and then make a plot showing the data points and all three curves on the same axes. (You will probably have to read the help page on Spline Continuity and End Conditions to see how to adjust the derivatives at the endpoints.)

11. (*expires 2/23*) Consider the set of six points (defined as `CSNY` on this page)

$$(-3.1, -1.94), (-2.2, 3.82), (-0.95, -4.05), (0.8, -0.05), (1.3, -0.221), (4.7, -1.30).$$

Find the function of the form

$$f(x) = a\cos(x) + b\sin(x) + k\cos(2x) + d\sin(2x) + g\cos(3x) + h\sin(3x)$$

with appropriate values of the constants a, b, k, d, g, h (correct to at least 6 significant figures) so that $f(x)$ passes through the given points.

Then plot the points and $f(x)$ on the same set of axes for $-2\pi < x < 2\pi$.

12. (*expires 2/23*) On the Wikipedia page on interpolation is an example of a cubic spline interpolating several points on an epitrochoid. An epitrochoid can be written in parametric form as $\{x(t) = (a+b)\cos(t) - c\cos\left((a/b+1)t\right),\ y(t) = (a+b)\sin(t) - c\sin\left((a/b+1)t\right)\}$. The file epidata.txt defines a list of points `epipts` on an epitrochoid, evaluated at each of the corresponding t-values given in the list `tvals`.

Use `CurveFitting[Spline]` to determine the cubic splines interpolating the given points, and then produce a plot showing both the interpolating curve and the points, analogous to the one on Wikipedia. Plot your spline for $0 \le t \le 4\pi$.

It is irrelevant for doing the problem, but the epitrochoid used has $a = 5$, $b = 2$, and $c = 5$.