April 4, 2024.

The last phugoid day...

> $with(DEtools):$

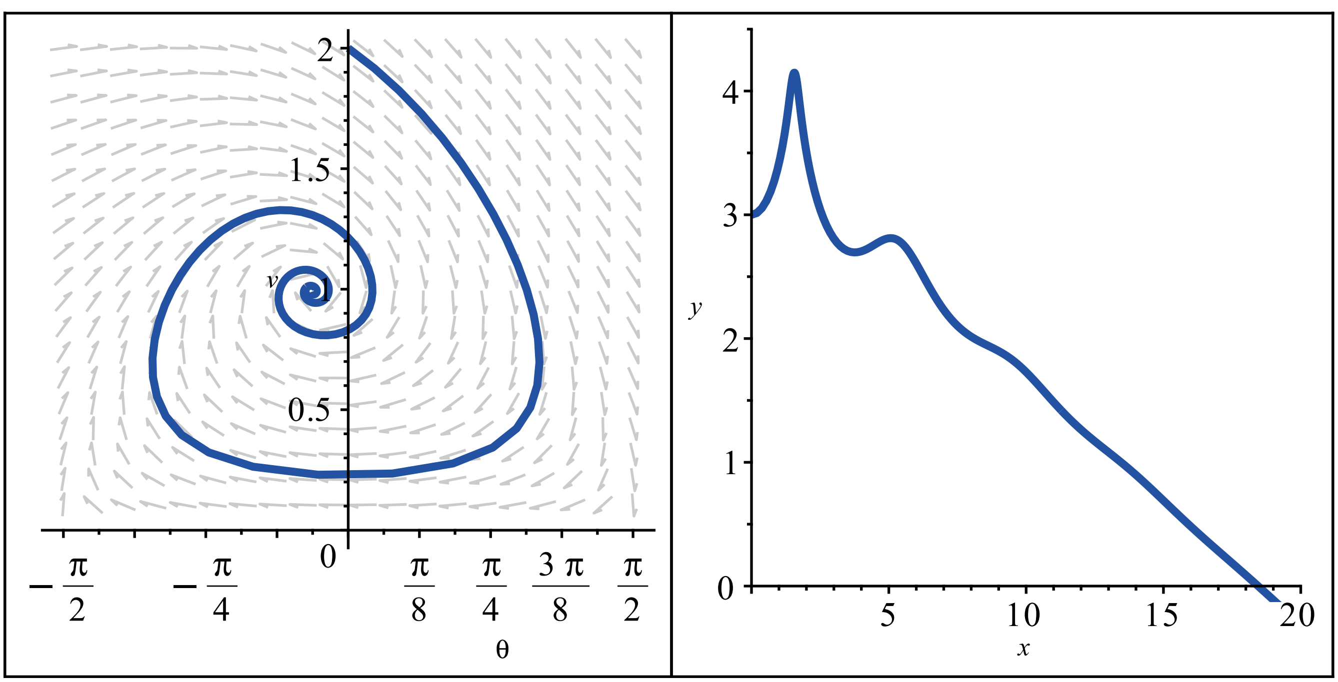
$$phug(R) := \left[diff(\text{theta}(t), t) = v(t) - \frac{\cos(\text{theta}(t))}{v(t)}, diff(v(t), t) = -\sin(\text{theta}(t)) - R \cdot v(t)^2 \right]:$$

Recall that previously we augmented this system to solve for the path of the glider, as well.

> $xphug(R) := [op(phug(R)), diff(x(t), t) = v(t) \cdot \cos(\text{theta}(t)), diff(y(t), t) = v(t) \cdot \sin(\text{theta}(t))]:$

```
> ShowPic:=proc(R, v0)
   local step:=.1;
   plots[display]( <
     DEplot( phug(R), [theta,v], t=0..30, [[theta(0)=0, v(0)=v0]],
            theta=-Pi/2..Pi/2, v=0..2, tickmarks=[piticks,default],
  color=gray,
            stepsize=step) |
     DEplot( xphug(R),[theta,v, x,y],t=0..30,
            [[theta(0)=0, v(0)=v0, x(0)=0, y(0)=3]],
            theta=-Pi/2..Pi/2, v=0..2,x=0..20, y=0..4.5, stepsize=
  step, scene=[x,y],
            obsrange=false)
    >);
  end:
```

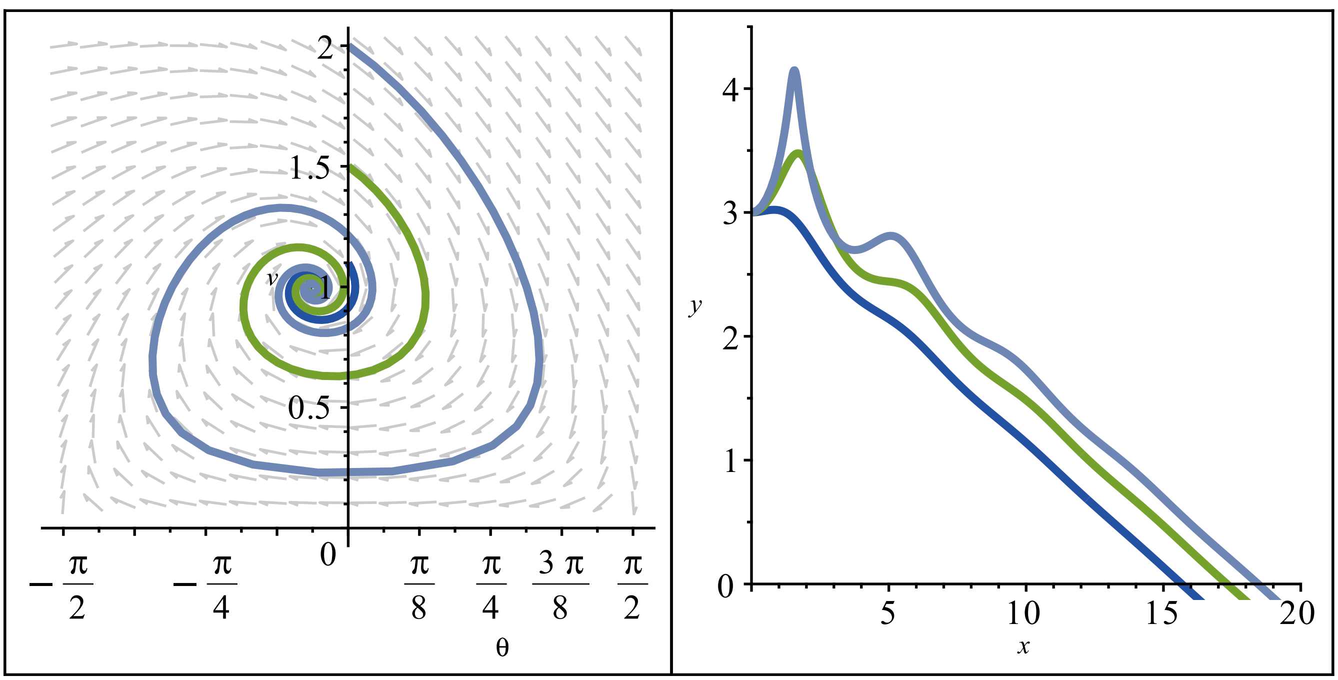
```
> ShowPic(.2,2);
```



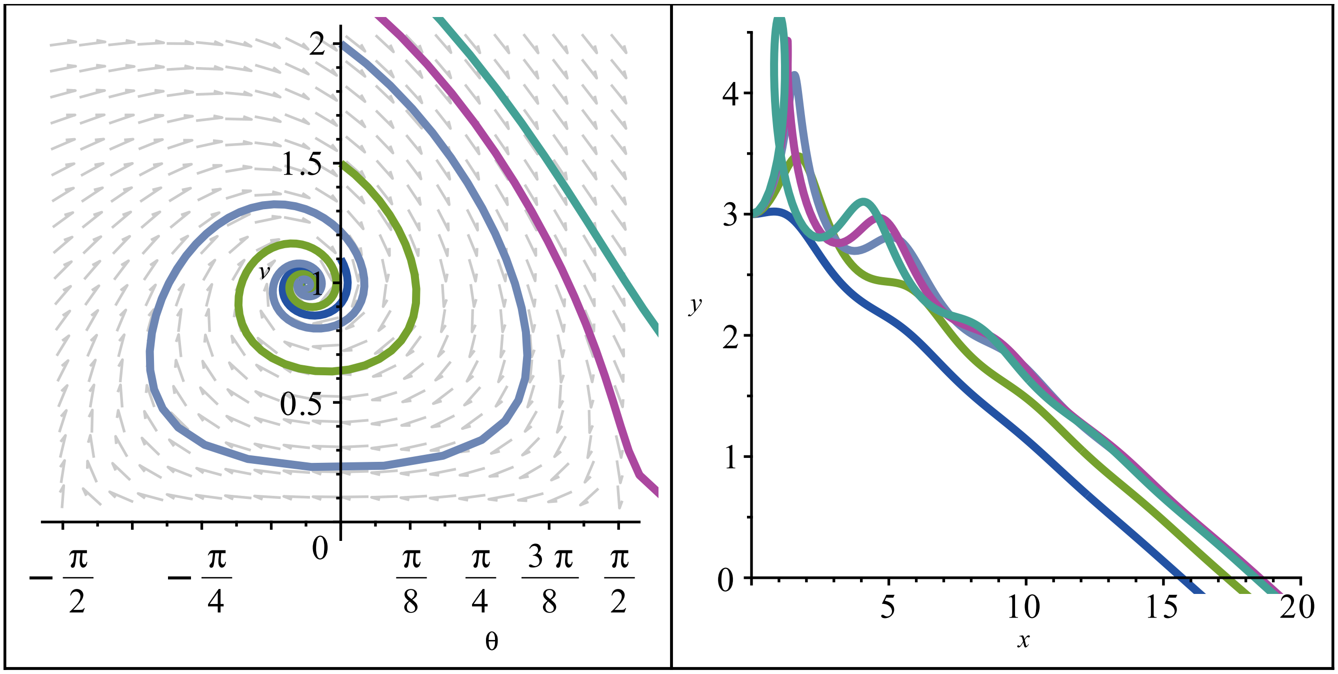
I'd prefer to see a couple of initial velocities, like ShowPic(.2, [1,1.5, 2])

```
> ShowPic:=proc(R, vlist)
   local step:=.1;
   local inits, xinits, i;
   inits:=[seq([theta(0)=0, v(0)=vlist[i]], i=1..nops(vlist))];
   xinits:=map(ini->[op(ini), x(0)=0, y(0)=3], inits);
   plots[display]( <
     DEplot( phug(R), [theta,v], t=0..30, inits,
            theta=-Pi/2..Pi/2, v=0..2, tickmarks=[piticks,default],
  color=gray,
            stepsize=step, obsrange=false) |
     DEplot( xphug(R),[theta,v, x,y],t=0..30,
            xinits,
            theta=-Pi/2..Pi/2, v=0..2,x=0..20, y=0..4.5, stepsize=
  step,
            scene=[x,y],
            obsrange=false)
    >);
  end:
```

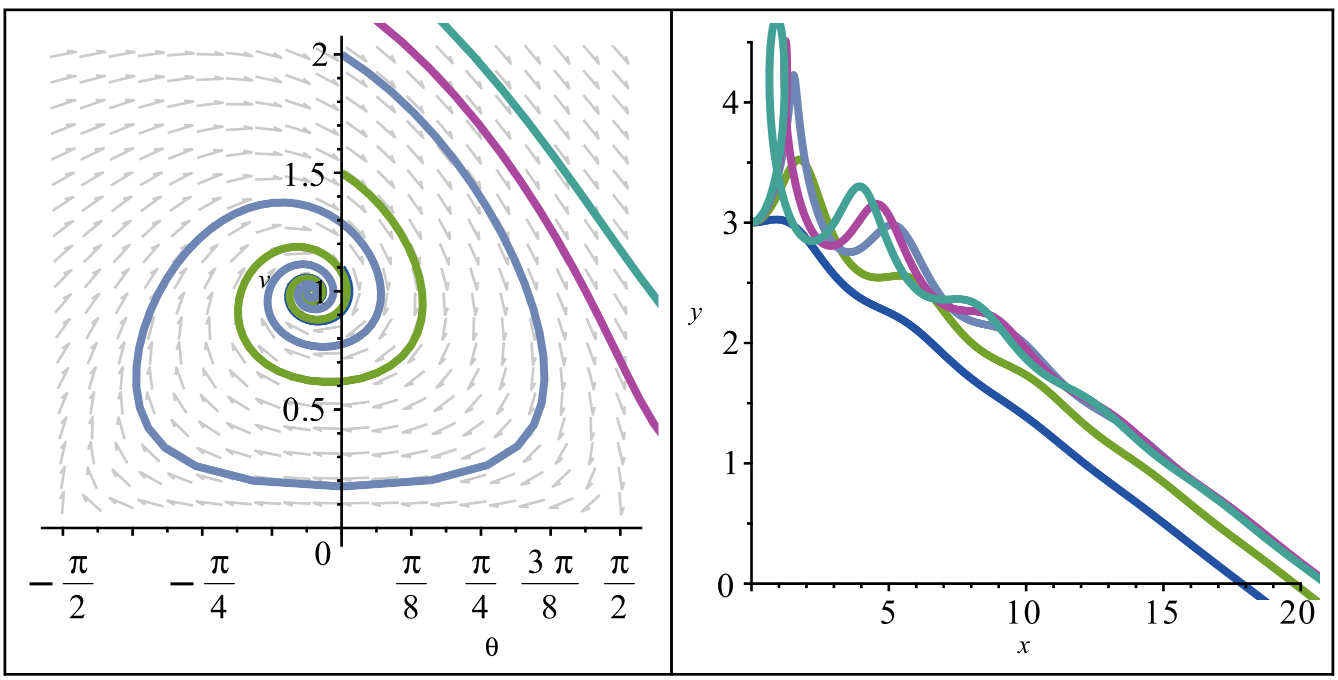
> *ShowPic*(.2, [1.1, 1.5, 2]);



> *ShowPic*(.2, [1.1, 1.5, 2, 2.25, 2.5]);




```
> ShowPic(.175, [1.1, 1.5, 2, 2.25, 2.5]);
```

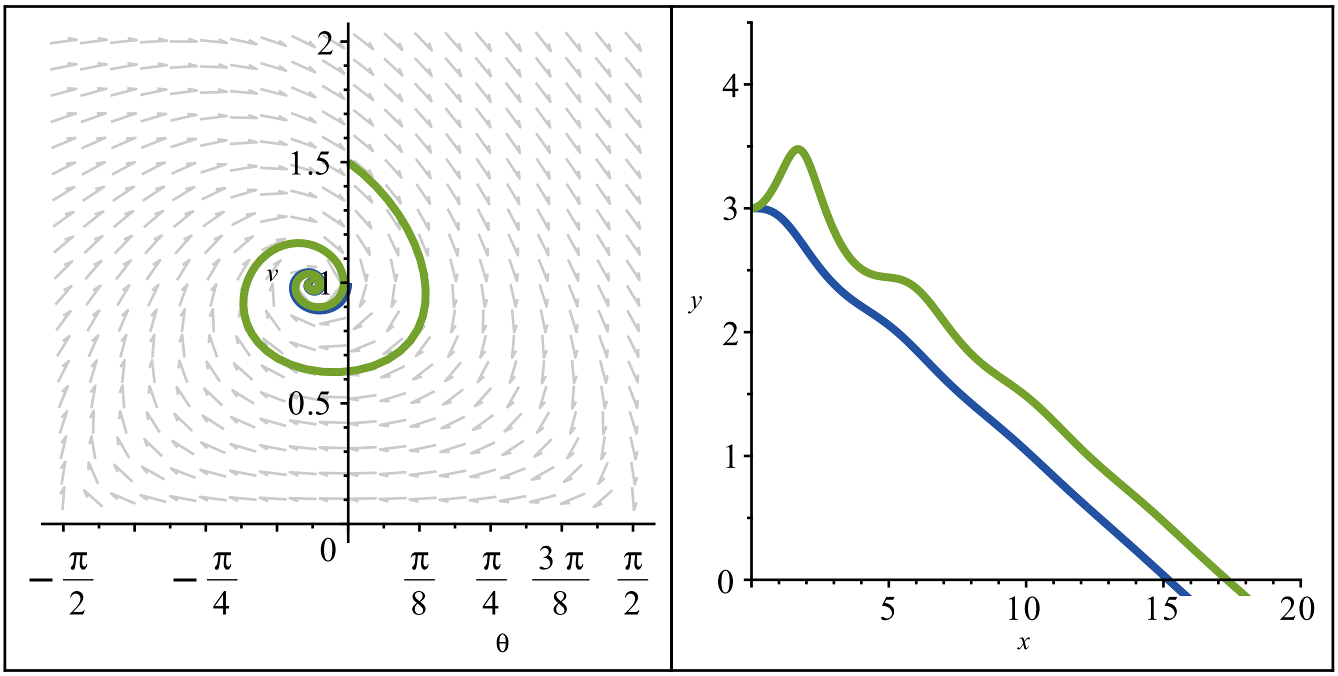


Does varying the initial angle make it go further?

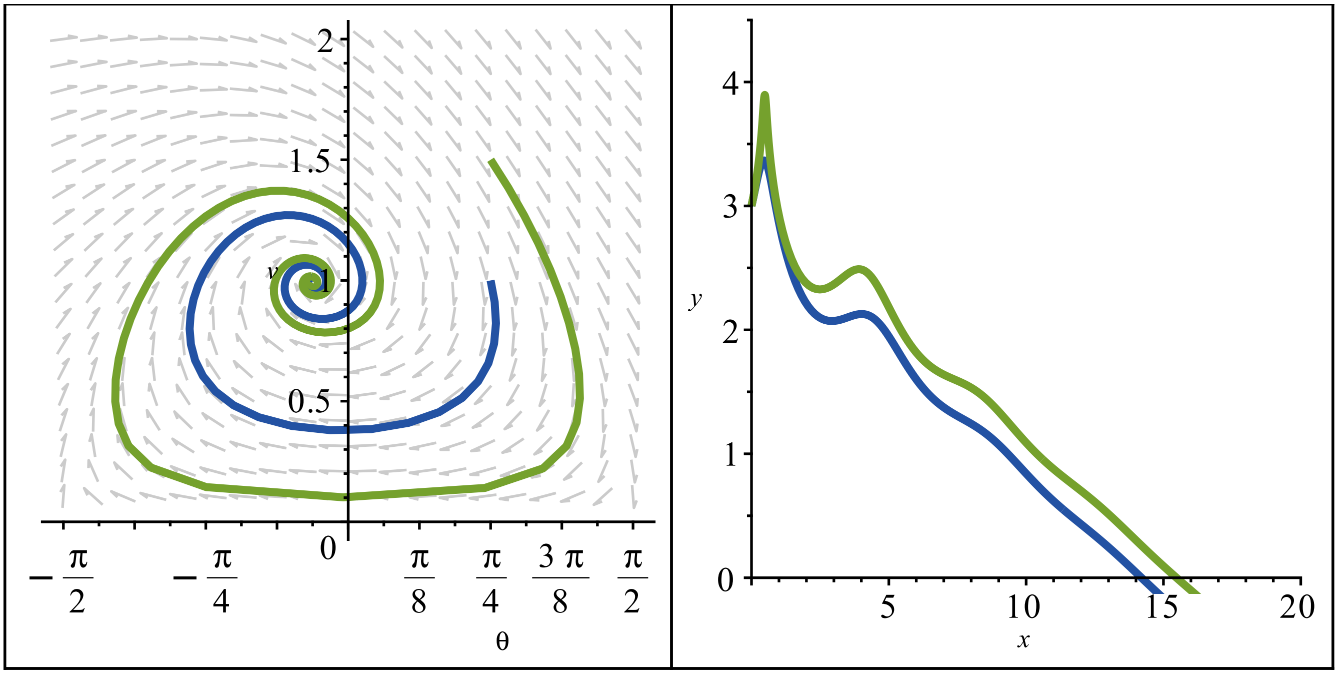
```
> ShowPic:=proc(R, vlist, theta0:=0)
   local step:=.1;
   local inits, xinits, i;
   inits:=[seq([theta(0)=theta0, v(0)=vlist[i]], i=1..nops(vlist))];
   xinits:=map(ini->[op(ini), x(0)=0, y(0)=3], inits);
   plots[display]( <
     DEplot( phug(R), [theta,v], t=0..30, inits,
            theta=-Pi/2..Pi/2, v=0..2, tickmarks=[piticks,default],
```

```
color=gray,
          stepsize=step, obsrange=false) |
   DEplot( xphug(R),[theta,v, x,y],t=0..30,
          xinits,
          theta=-Pi/2..Pi/2, v=0..2,x=0..20, y=0..4.5, stepsize=
step,
          scene=[x,y],
          obsrange=false)
  >);
end:
```

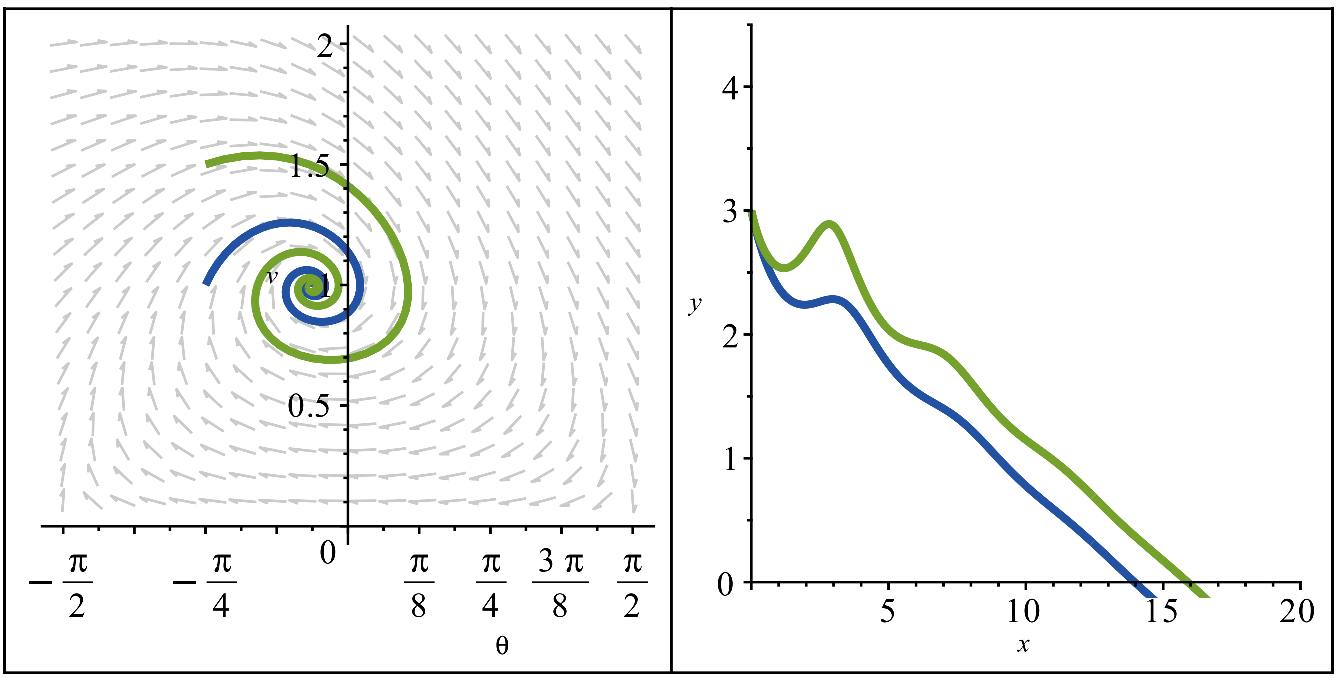
> $ShowPic(.2, [1, 1.5]);$



> $ShowPic\left(.2, [1, 1.5], \frac{\text{Pi}}{4}\right);$



$> ShowPic\left(.2, [1, 1.5], \frac{-\mathrm{Pi}}{4}\right);$



Let's try to answer the question:

Fixing R=0.2, theta(0)=0, and y(0)=3, what is the largest x(T) I can get where y(T)=0 by varying v(0)?
ie, maximize x(t_hit) as a function of v(0).

use dsolve(numeric)

$> sol0 := dsolve(\{op(xphug(.2)), \mathrm{theta}(0) = 0, x(0) = 0, y(0) = 3, v(0) = 2\}, numeric, stepsize = .1)$

(1)

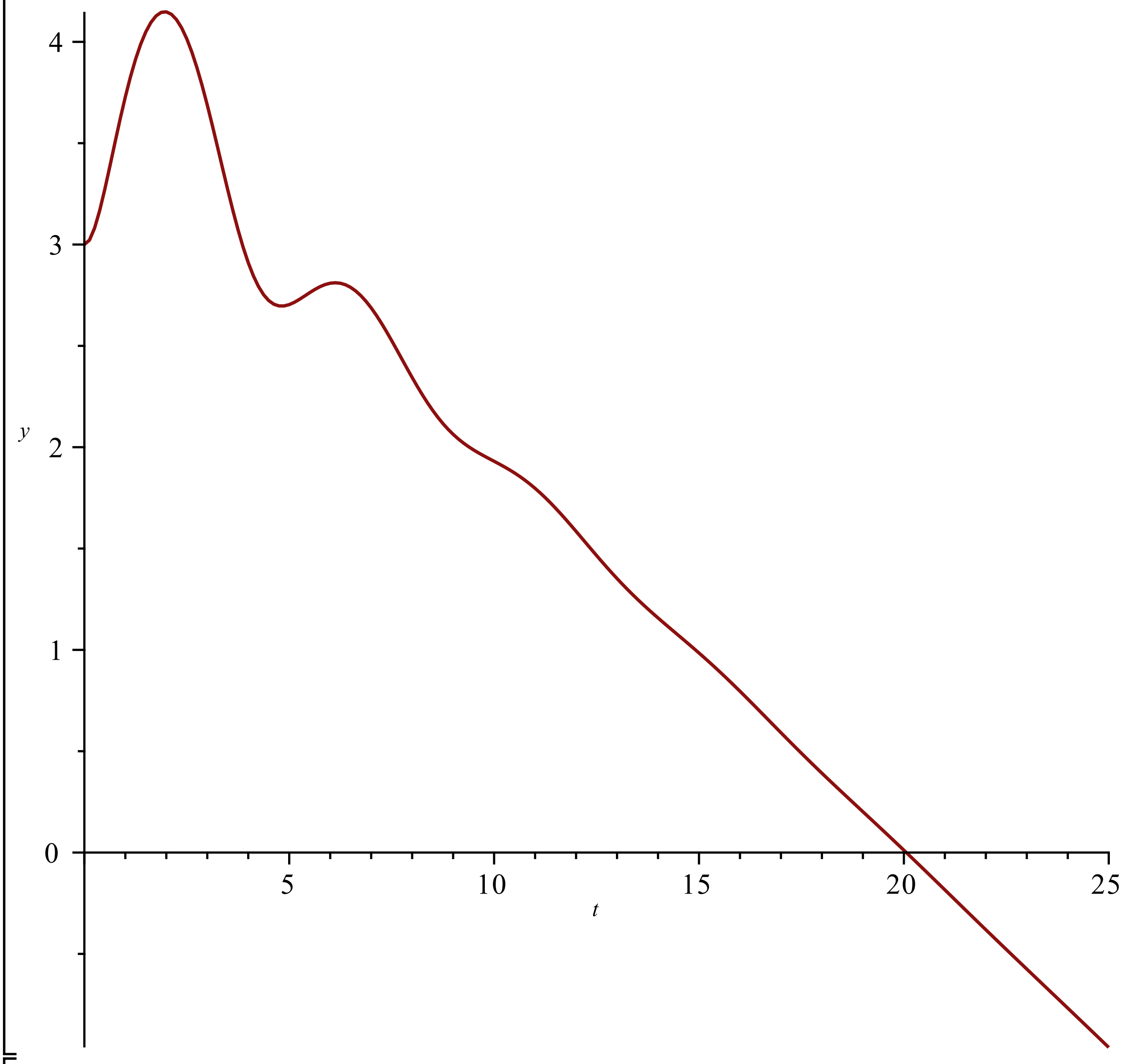
$$sol0 := \textbf{proc}(x\_rkf45) \; ... \; \textbf{end proc} \qquad \textbf{(1)}$$

> *sol0*(20);

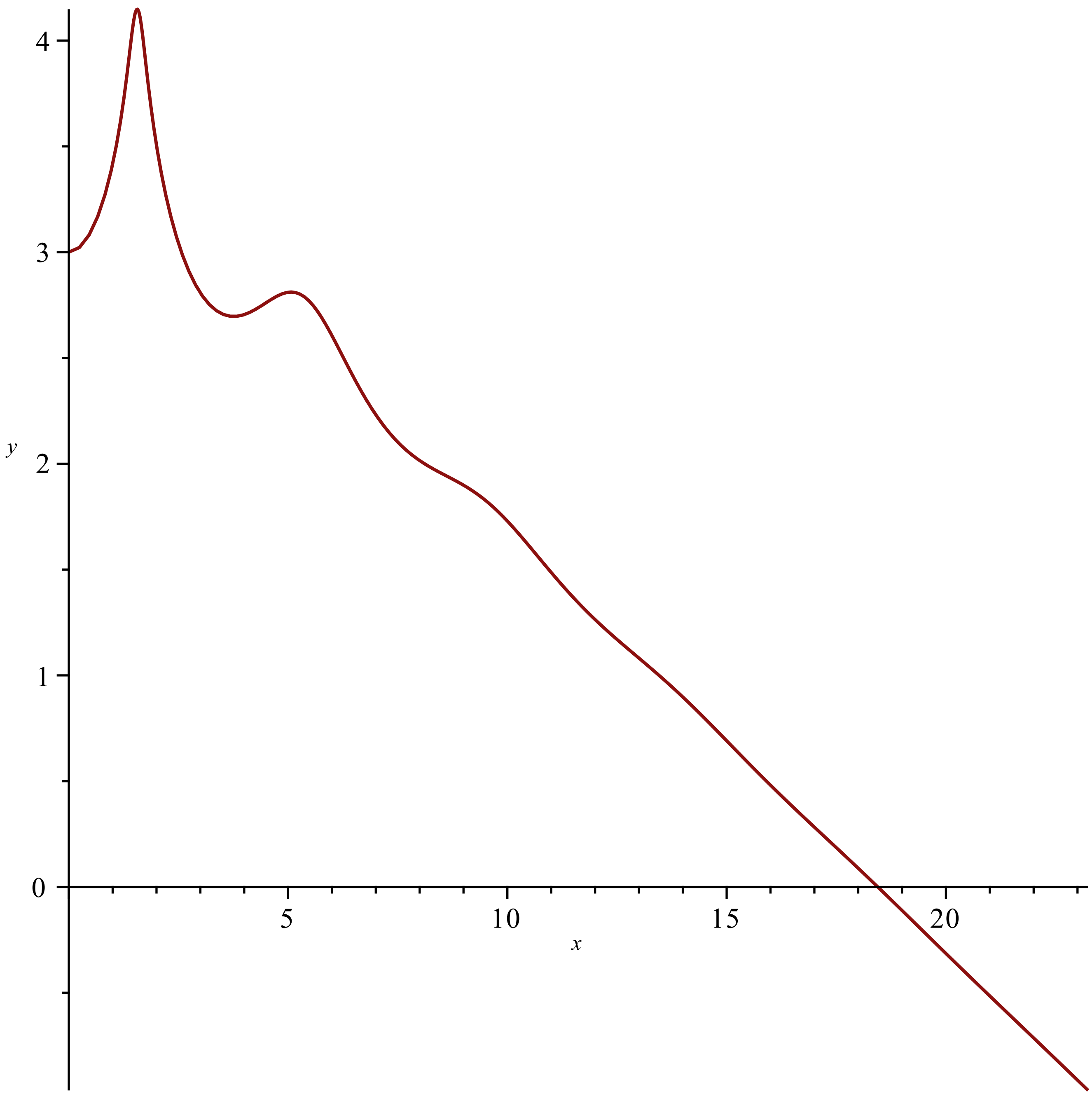
$$[t = 20., \theta(t) = -0.196604734090982, v(t) = 0.987028563714960, x(t) = 18.3888929766207, y(t) = 0.0114994944851027] \qquad \textbf{(2)}$$

> *with*(*plots*) :

> *C*

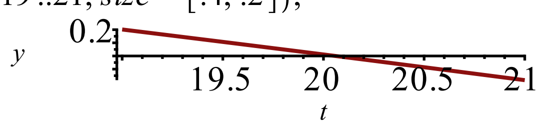


> *odeplot*(*sol0*, [*x*(*t*), *y*(*t*)], *t* = 0 ..25);

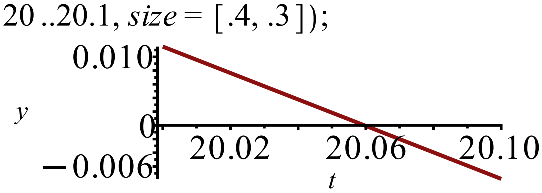


How to find the time? Graphically is slow, but easier to explain.

> *odeplot*(*sol0*, [*t*, *y*(*t*)], *t* = 19 ..21, *size* = [.4, .2]);



> *odeplot*(*sol0*, [*t*, *y*(*t*)], *t* = 20 ..20.1, *size* = [.4, .3]);



One way to see when the plane hit the ground is to add a new variable to the system that stops when y(t)

<0.

> $xphug(R);$

$$\left[\frac{d}{dt}\,\theta(t) = v(t) - \frac{\cos(\theta(t))}{v(t)},\ \frac{d}{dt}\,v(t) = -\sin(\theta(t)) - R\,v(t)^2,\ \frac{d}{dt}\,x(t) = v(t)\cos(\theta(t)),\ \frac{d}{dt}\,y(t) = v(t)\sin(\theta(t))\right] \quad (3)$$

> $tphug := R \rightarrow [op(xphug(R)), diff(H(t), t) = piecewise(y(t) > 0, 1, 0)];$

$$tphug := R \mapsto \left[op(xphug(R)),\ \frac{d}{dt}\,H(t) = \begin{cases} 1 & 0 < y(t) \\ 0 & otherwise \end{cases}\right] \quad (4)$$

> $solT := dsolve(\{op(tphug(.2)), H(0) = 0, \text{theta}(0) = 0, x(0) = 0, y(0) = 3, v(0) = 2\}, numeric, stepsize = .1)$

$$solT := \textbf{proc}(x\_rkf45)\ \ldots\ \textbf{end proc} \quad (5)$$

> $solT(21);$

$$[t = 21.,\ H(t) = 20.0673868509658,\ \theta(t) = -0.200622003946283,\ v(t) = 0.989590952303004,\ x(t) = 19.3574306819860,\ y(t) = -0.183956648323318] \quad (6)$$

>