April 2, 2024

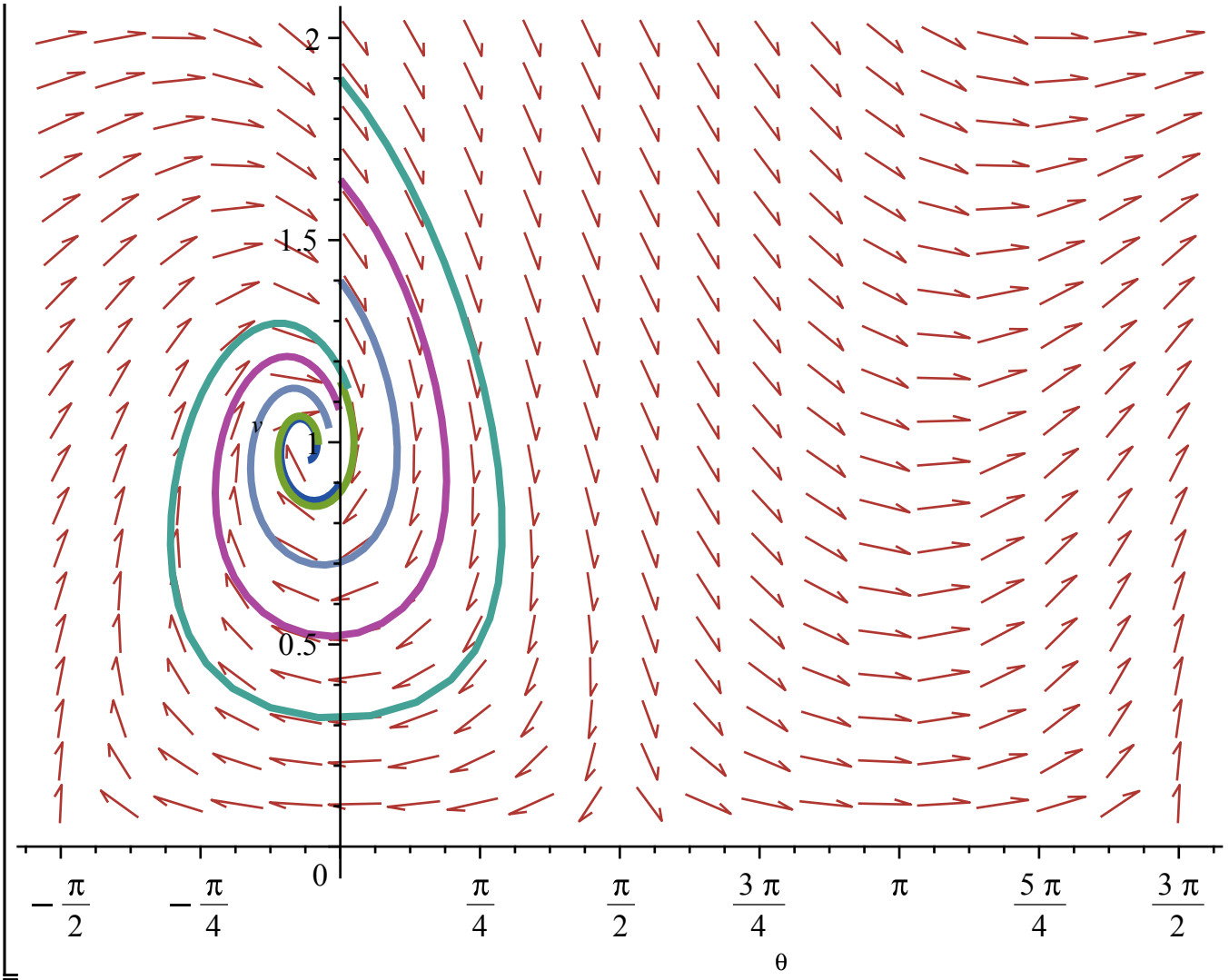
```
> R := `R`;
  phug := [diff(theta(t), t) = v(t) - cos(theta(t))/v(t), diff(v(t), t) = -sin(theta(t)) - R*v(t)^2];
```

$$R := 0.2$$

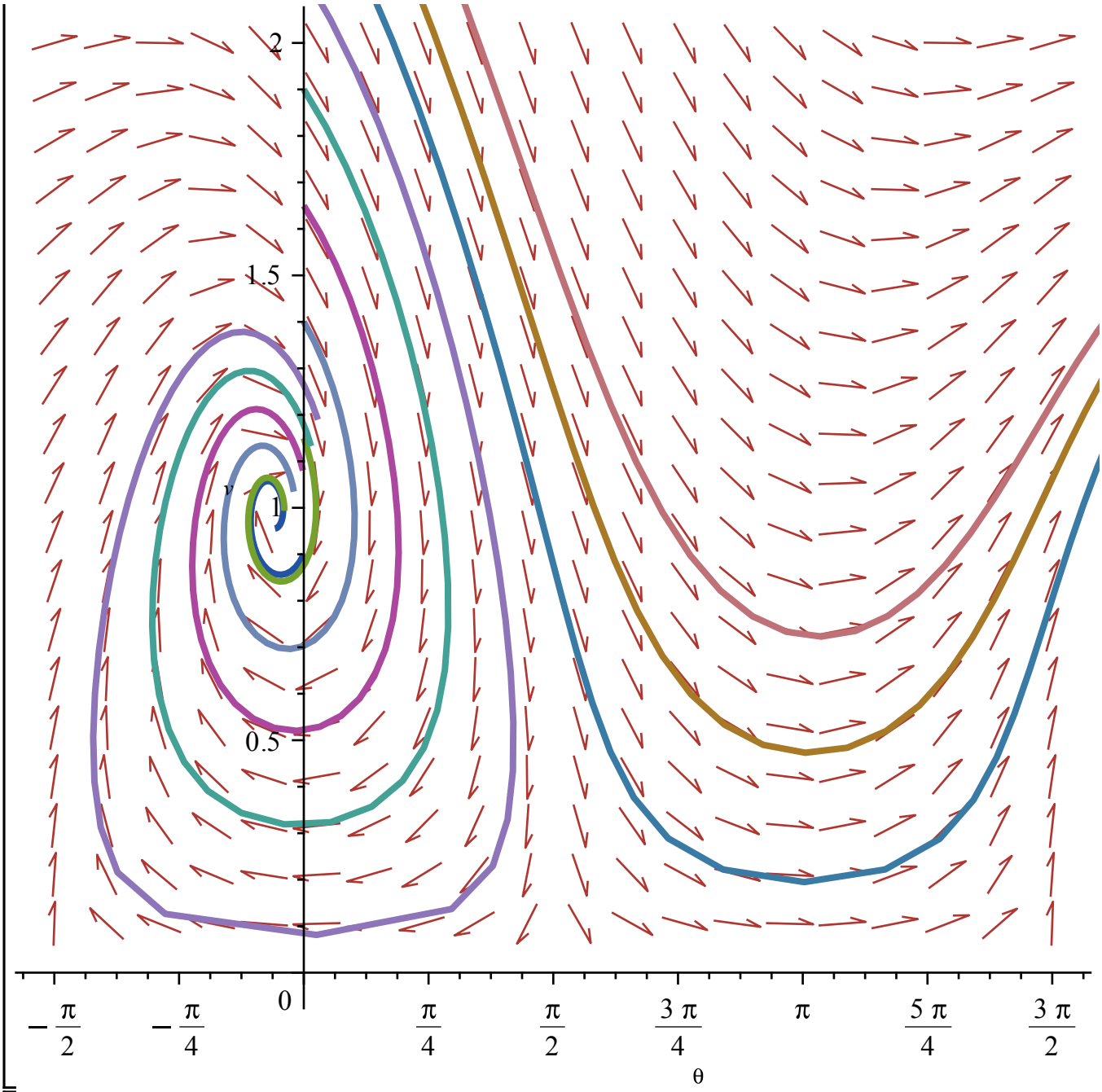
$$phug := \left[\frac{d}{dt}\,\theta(t) = v(t) - \frac{\cos(\theta(t))}{v(t)}, \frac{d}{dt}\,v(t) = -\sin(\theta(t)) - 0.2\,v(t)^2\right] \quad (1)$$

```
> with(DEtools):
> R := 0.2;
  DEplot(phug, [theta, v], t = 0..5,
    [seq([v(0) = j, theta(0) = 0], j = 0.9..3.0, .25)], # generate initial conditions
    theta = -Pi/2..3*Pi/2, v = 0..2, tickmarks = [piticks, default]);
```

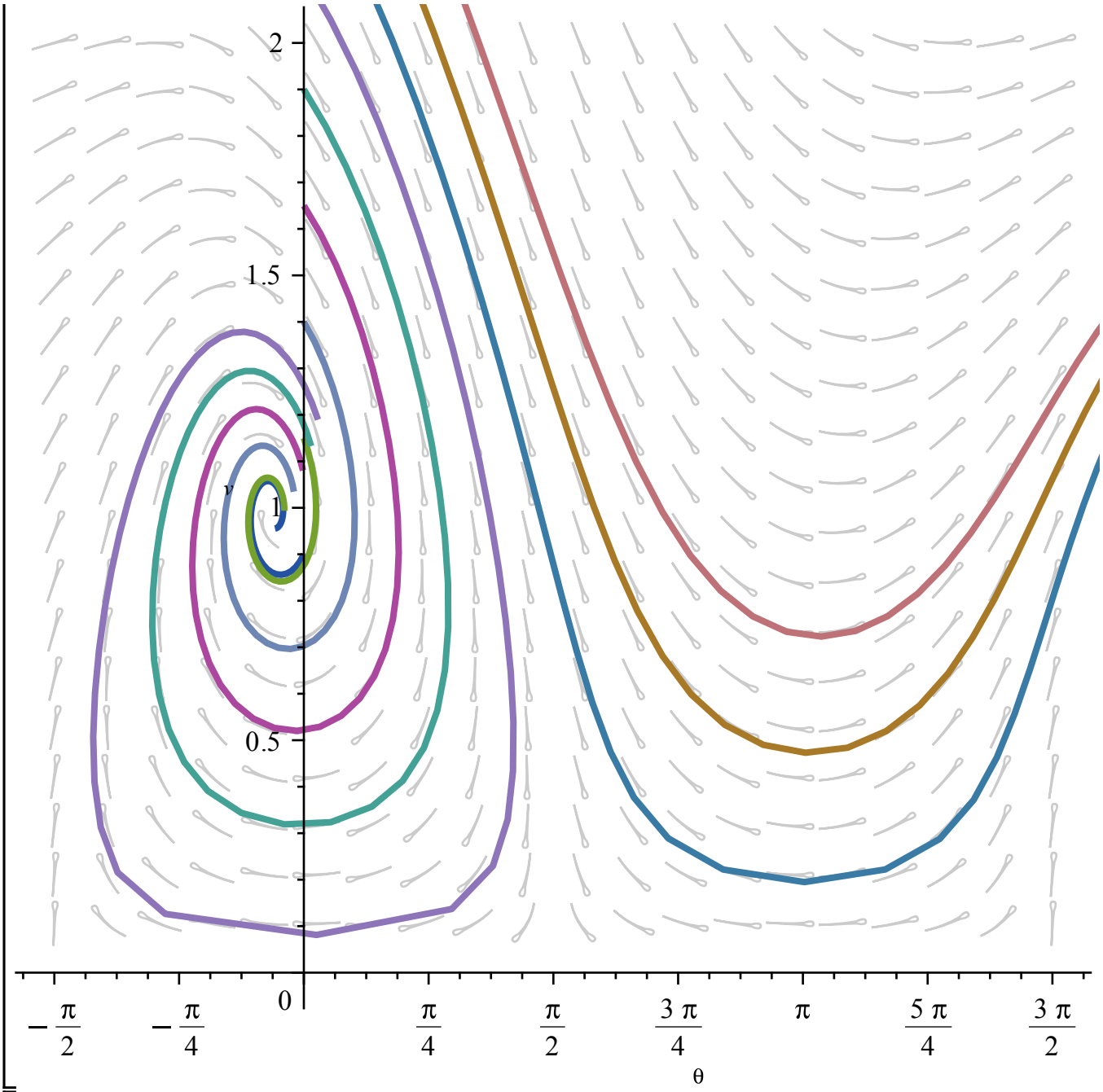
$$R := 0.2$$



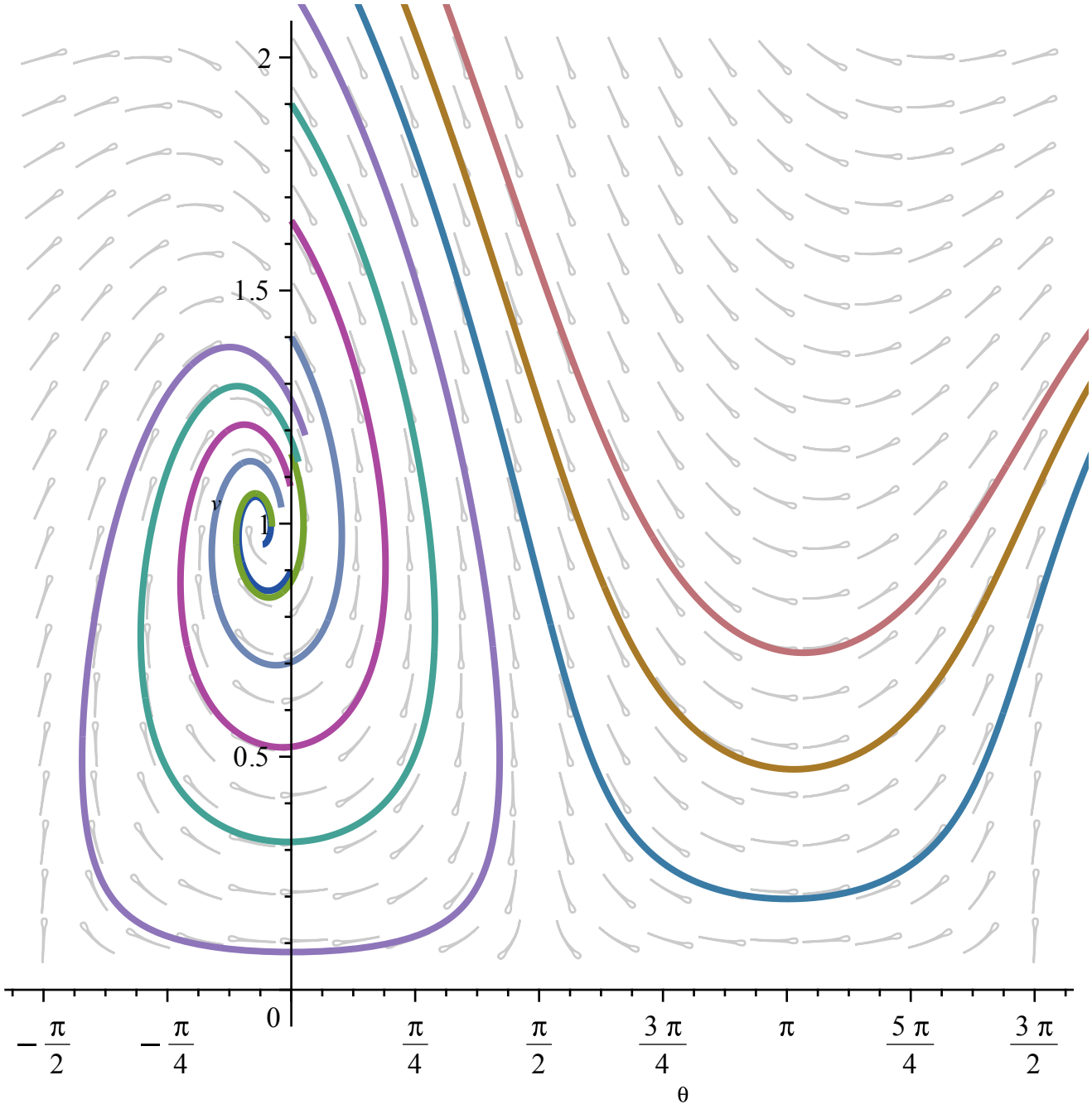

```
> DEplot( phug, [theta, v], t = 0..5,
    [seq([v(0) = j, theta(0) = 0], j = 0.9..3.0, .25)],   # generate initial conditions
    theta = -Pi/2..(3*Pi)/2, v = 0..2, tickmarks = [piticks, default], obsrange = false );
```



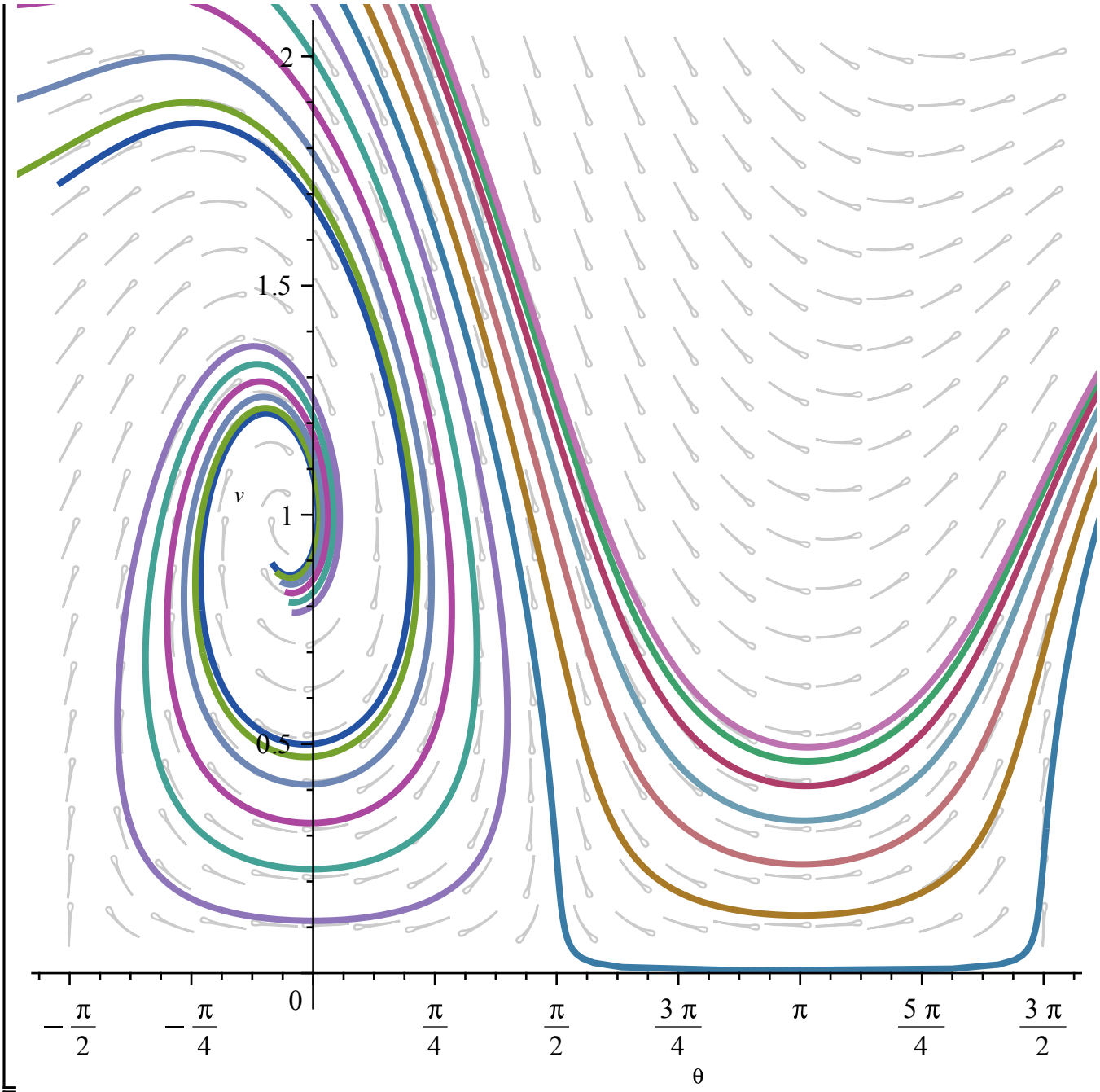

```
> DEplot( phug, [theta, v], t = 0..5,
    [seq( [v(0) = j, theta(0) = 0], j = 0.9..3.0, .25 )],   # generate initial conditions
  theta = -Pi/2..3·Pi/2, v = 0..2, tickmarks = [piticks, default], obsrange = false,
  color = grey, arrows = comet );
```



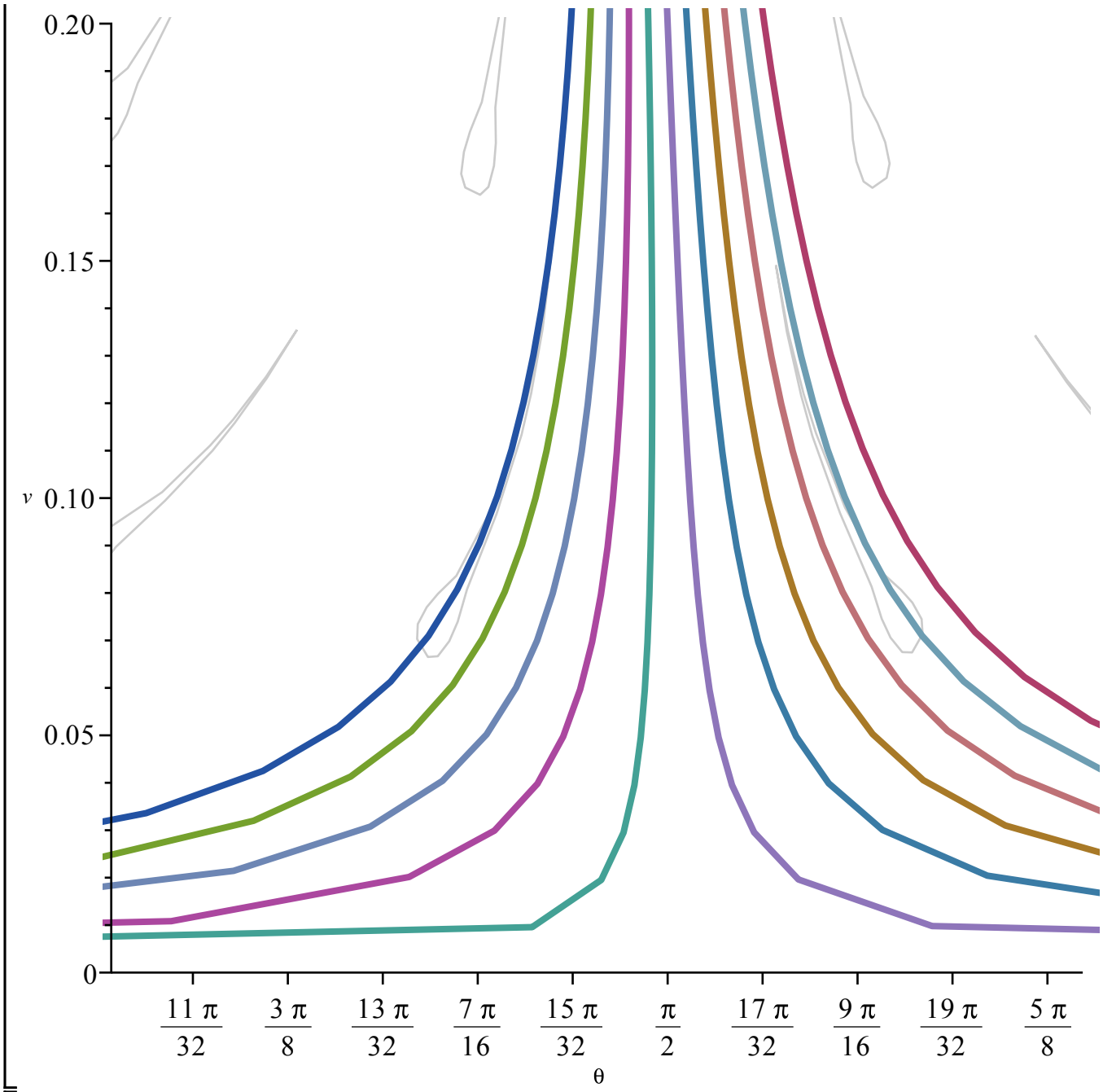

```
> DEplot( phug, [theta, v], t = 0..5,
    [seq( [v(0) = j, theta(0) = 0], j = 0.9..3.0, .25 )],   # generate initial conditions
  theta = -Pi/2..3*Pi/2, v = 0..2, tickmarks = [piticks, default], obsrange = false,
  color = grey, arrows = comet, stepsize = 0.01 );
```




```
> DEplot( phug, [theta, v], t=-3..5,
  [seq([v(0)=0.5, theta(0)=th], th=0..3.2, .25)], # generate initial conditions
  theta=-Pi/2..3*Pi/2, v=0..2, tickmarks=[piticks, default], obsrange=false,
  color=grey, arrows=comet, stepsize=0.01);
```




```
> DEplot(phug, [theta, v], t=-3..5,
    [seq([v(0)=0.2, theta(0)=th], th=Pi/2-0.1..Pi/2+.1, 0.02)],
    # generate initial conditions
  theta=-Pi/2..3*Pi/2, v=0..2, tickmarks=[piticks, default], obsrange=false,
  color=grey, arrows=comet, stepsize=0.01, view=[1..2, 0..0.2]);
```



> *phug*

$$\left[\frac{d}{dt}\,\theta(t) = v(t) - \frac{\cos(\theta(t))}{v(t)},\ \frac{d}{dt}\,v(t) = -\sin(\theta(t)) - 0.2\,v(t)^2\right] \quad \textbf{(2)}$$

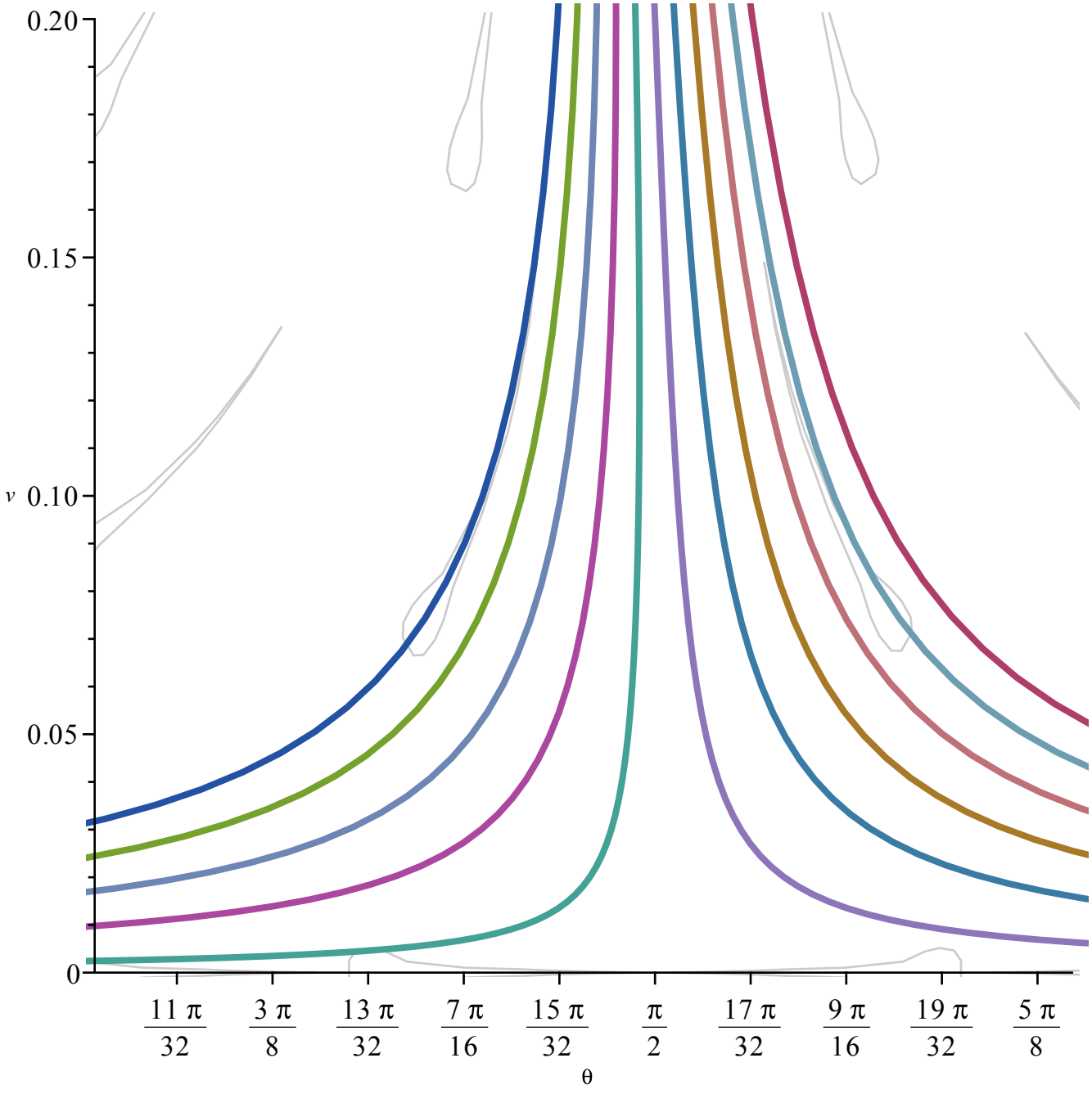
> *vphug* := [*diff*(theta(*t*), *t*) = *v*(*t*)^2 − cos(theta(*t*)), *diff*(*v*(*t*), *t*) = *v*(*t*)·(−sin(theta(*t*)) − *R* ·*v*(*t*)^2)]

$$vphug := \left[\frac{d}{dt}\,\theta(t) = v(t)^2 - \cos(\theta(t)),\ \frac{d}{dt}\,v(t) = v(t)\,\left(-\sin(\theta(t)) - 0.2\,v(t)^2\right)\right] \quad \textbf{(3)}$$

> *DEplot*(*vphug*, [theta, *v*], *t* = −1 ..5,

[*seq*([*v*(0) = 0.2, theta(0) = *th*], *th* = Pi/2 − 0.1 .. Pi/2 + .1, 0.02)],

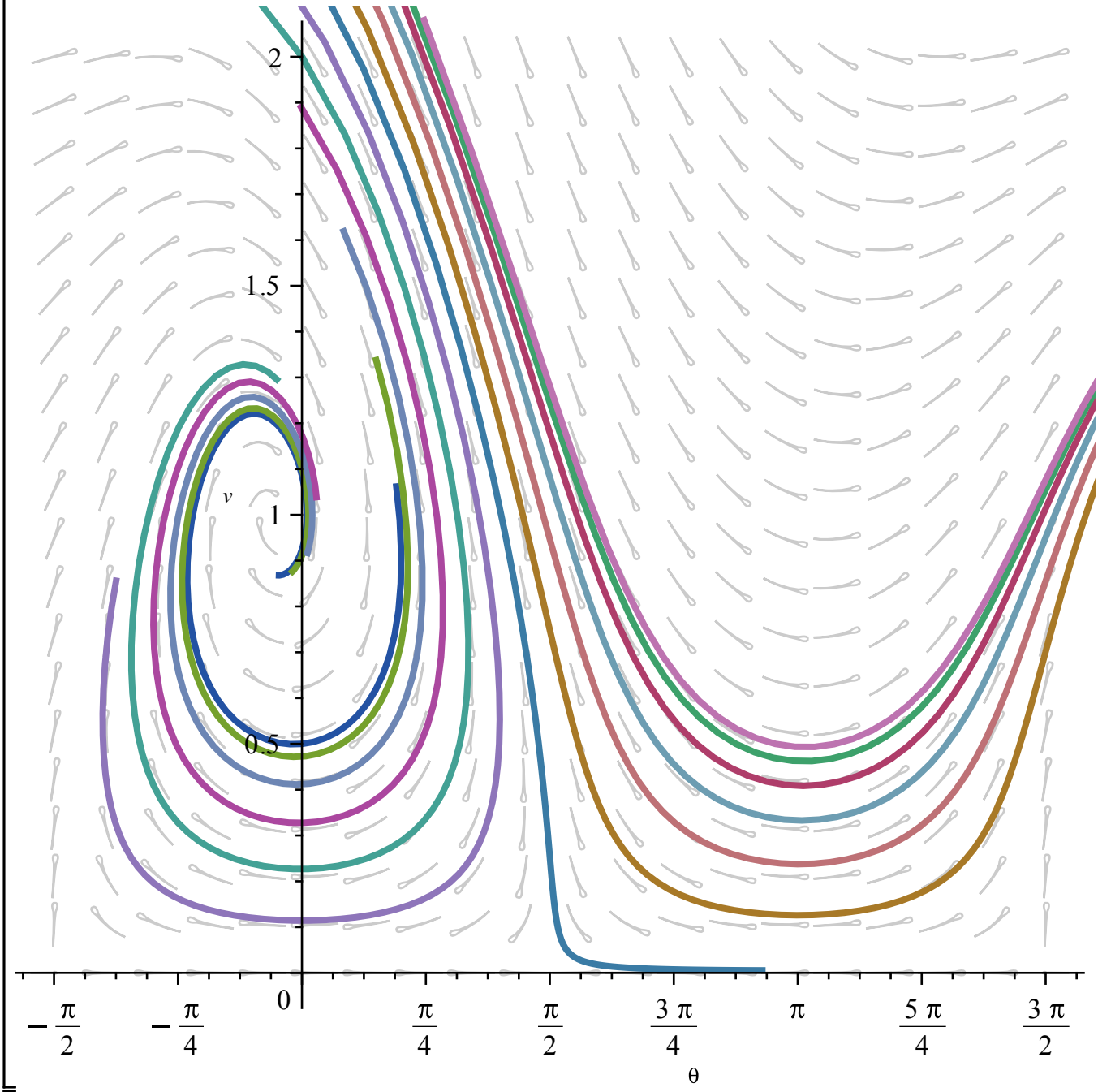
```
# generate initial conditions
theta = -Pi/2 .. 3*Pi/2, v = 0 ..2, tickmarks = [piticks, default], obsrange = false,
color = grey, arrows = comet, stepsize = 0.1, view = [1 ..2, 0 ..0.2 ]);
```




```
> DEplot( vphug, [theta, v], t = -1.5 ..5,
  [seq( [v(0) = 0.5, theta(0) = th], th = 0 ..3.2, .25 ) ],   # generate initial conditions
```

theta = $-\frac{Pi}{2}$ .. $\frac{3 \cdot Pi}{2}$, $v = 0..2$, *tickmarks* = [*piticks*, *default*], *obsrange* = *false*,

color = *grey*, *arrows* = *comet*, *stepsize* = 0.1);



Want jacobian at π/2, 0...

> *vphug*

$$\left[\frac{d}{dt}\theta(t) = v(t)^2 - \cos(\theta(t)),\ \frac{d}{dt}v(t) = v(t)\left(-\sin(\theta(t)) - 0.2\,v(t)^2\right)\right] \quad (4)$$

> *map*(*rhs*, *vphug*);

$$\left[v(t)^2 - \cos(\theta(t)),\ v(t)\left(-\sin(\theta(t)) - 0.2\,v(t)^2\right)\right] \quad (5)$$

```
> subs( {v(t) = v, theta(t) = theta}, map(rhs, vphug) );
```

$$[v^2 - \cos(\theta),\ v\,(-\sin(\theta) - 0.2\,v^2)] \tag{6}$$

```
> F := unapply( subs( {v(t) = v, theta(t) = theta}, map(rhs, vphug) ), [theta, v]);
```

$$F := (\theta, v) \mapsto [v^2 - \cos(\theta),\ v\cdot(-\sin(\theta) - 0.2\cdot v^2)] \tag{7}$$

```
> F(v, theta)
```

$$[\theta^2 - \cos(v),\ \theta\,(-\sin(v) - 0.2\,\theta^2)] \tag{8}$$

```
> solve(F(theta, v), { theta, v});
```

$$\{\theta = 1.570796327, v = 0.\}, \{\theta = -0.1973955598, v = 0.9902427357\}, \{\theta = -0.1973955598, v = -0.9902427357\}, \{\theta = 2.944197094, v = 0.9902427357\,I\}, \{\theta = 2.944197094, v = -0.9902427357\,I\} \tag{9}$$

Let's find the Jacobian at (Pi/2, 0)

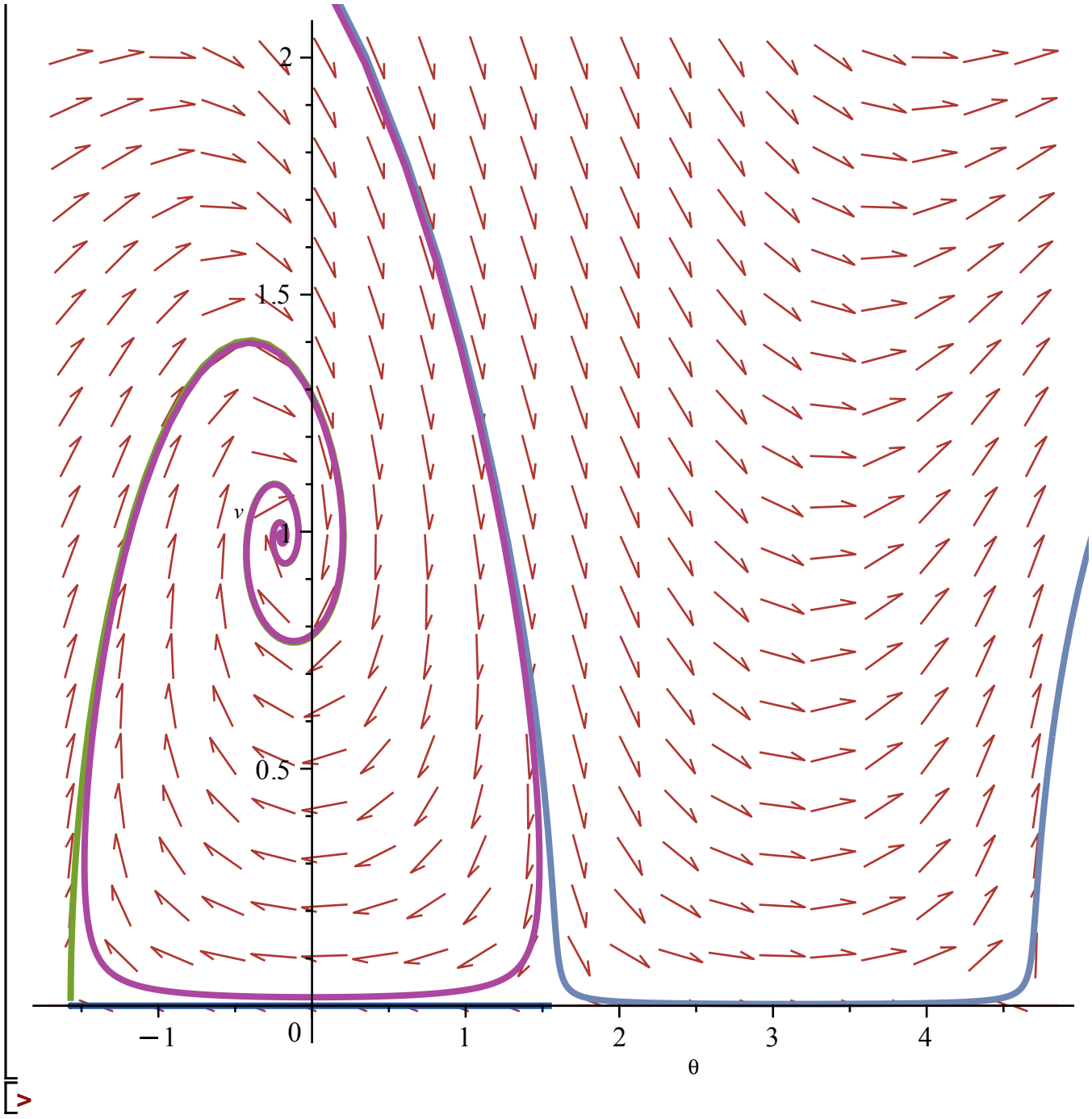
```
> with(LinearAlgebra) :
  with(VectorCalculus) :
> Jack := Jacobian(F(theta, v), [theta, v])
```

$$Jack := \begin{bmatrix} \sin(\theta) & 2\,v \\ -v\cos(\theta) & -\sin(\theta) - 0.6\,v^2 \end{bmatrix} \tag{10}$$

```
> eval( Jack, { theta = Pi/2, v = 0 } )
```

$$\begin{bmatrix} 1 & 0 \\ 0 & -1. \end{bmatrix} \tag{11}$$

```
> DEplot( vphug, [theta, v], t = 0..50,
  [[v(0) = 0, theta(0) = Pi/2 - 0.01], [v(0) = 0.01, theta(0) = -Pi/2],
  [v(0) = 2.23, theta(0) = 0], [v(0) = 2.21, theta(0) = 0]],
  theta = -Pi/2 .. 3*Pi/2, v = 0..2, obsrange = false, stepsize = 0.1 );
```


2
1.5
v
1
0.5
−1
0
1
2
3
4
θ
v