6 February 2024

Last time, we left off with something we didn't actually get to. So let's start there.

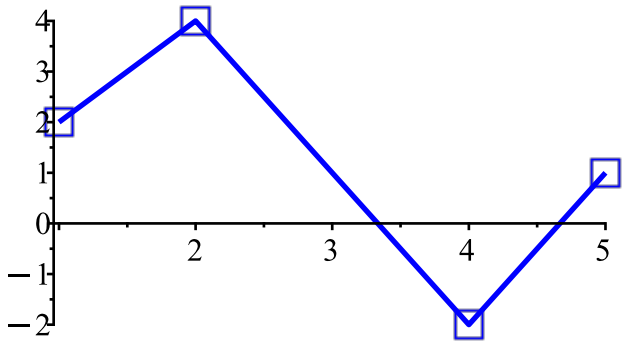
The goal here is the following:

Given some points, find the (unique) polynomial of highest degree that passes through all of them.

```
> pts := [ [1, 2], [2, 4], [4,-2], [5, 1] ];
```

$$pts := [[1, 2], [2, 4], [4, -2], [5, 1]] \quad (1)$$

```
> plot(pts, style=pointline,
      symbol=box, symbolsize=40, color=blue, thickness=2, size=[.5, "golden"])
```



It's a cubic, so

```
> cub:=x-> a+b*x+c*x^2+d*x^3;
```

$$cub := x \mapsto a + b \cdot x + c \cdot x^2 + d \cdot x^3 \quad (2)$$

```
> cub(1)=2
```

$$a + b + c + d = 2 \quad (3)$$

```
> #  solve(cub(1)=2,cub(2)=4, .... don't want to too much typing)
Error, unexpected single forward quote
> cub(pts[1][1])
```

$$a + b + c + d \quad (4)$$

```
> cub(pts[1][1])=pts[1][2];
```

$$a + b + c + d = 2 \quad (5)$$

Here I can get all the points plugged in to get the equations I want to solve.

```
> seq(cub(pts[i][1])=pts[i][2], i=1..nops(pts))
```

$$a + b + c + d = 2,\ a + 2\,b + 4\,c + 8\,d = 4,\ a + 4\,b + 16\,c + 64\,d = -2,\ a + 5\,b + 25\,c + 125\,d = 1 \quad (6)$$

```
> nops(pts)
```

$$4 \quad (7)$$

```
> eqns:=[seq(cub(pts[i][1])=pts[i][2], i=1..nops(pts))];
```

$$eqns := [a + b + c + d = 2,\ a + 2\,b + 4\,c + 8\,d = 4,\ a + 4\,b + 16\,c + 64\,d = -2,\ a + 5\,b + 25\,c + 125\,d = 1] \quad (8)$$

```
> solve([eqns[1],eqns[2],eqns[3]],{a,b,c})
```

$$\left\{a = -8\,d - \frac{10}{3},\ b = 14\,d + 7,\ c = -7\,d - \frac{5}{3}\right\} \tag{9}$$

```
> solve(eqns,{a,b,c,d})
```

$$\left\{a = -\frac{32}{3},\ b = \frac{119}{6},\ c = -\frac{97}{12},\ d = \frac{11}{12}\right\} \tag{10}$$

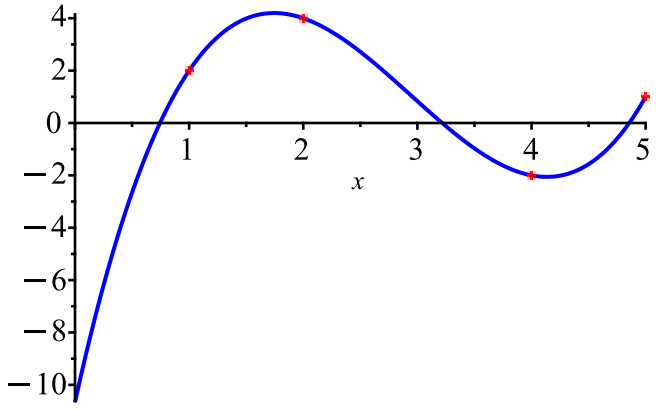
```
> subs(%,cub(x));
```

$$\frac{11}{12}x^3 - \frac{97}{12}x^2 + \frac{119}{6}x - \frac{32}{3} \tag{11}$$

```
> mycubic:=%;
```

$$mycubic := \frac{11}{12}x^3 - \frac{97}{12}x^2 + \frac{119}{6}x - \frac{32}{3} \tag{12}$$

```
> plot([mycubic, pts], x=0..5,  style=[line,point],
   color=[blue,red], size=[.5,"golden"]);
```



Maple has this already built in.

```
> with(CurveFitting);
```

[*ArrayInterpolation*, *BSpline*, *BSplineCurve*, *Interactive*, *LeastSquares*, *Lowess*, *PolynomialInterpolation*, *RationalInterpolation*, *Spline*, *ThieleInterpolation*] (13)

```
> PolynomialInterpolation(pts,x)
```

$$\frac{11}{12}x^3 - \frac{97}{12}x^2 + \frac{119}{6}x - \frac{32}{3} \tag{14}$$

Let's try this with more points -- I pasted this from daily/extras on the class web page wiggly.txt

```
> Dat := [
  [-3.0, 1.500718640],
  [-2.5, -1.333413875],
  [-2.0, -1.506526723],
  [-1.5,  .930868789],
  [-1.0, 3.678347877],
  [ -.5, 4.315138568],
  [ 0.0, 2.500000000],
  [  .5,  .225163738],
  [ 1.0, -.094494713],
  [ 1.5, 2.079138715],
  [ 2.0, 4.852883103],
  [ 2.5, 5.617076061],
```

```
  [ 3.0,  3.459451646]
  ] ;
```

$$Dat := [[-3.0, 1.500718640], [-2.5, -1.333413875], [-2.0, -1.506526723], [-1.5, 0.930868789], [-1.0, 3.678347877], [-0.5, 4.315138568], [0., 2.500000000], [0.5, 0.225163738], [1.0, -0.094494713], [1.5, 2.079138715], [2.0, 4.852883103], [2.5, 5.617076061], [3.0, 3.459451646]] \quad (15)$$

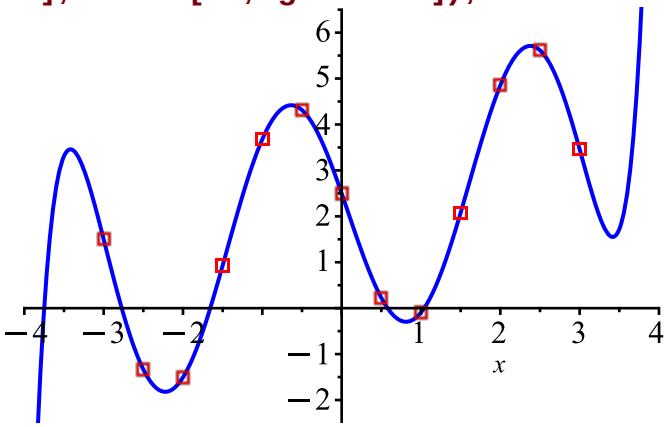
```
> nops(Dat);
```

$$13 \quad (16)$$

```
> poly:=PolynomialInterpolation(Dat,x)
```

$$poly := 2.582474908 \times 10^{-6} x^{12} + 0.00008506225844 x^{11} - 0.0001281972969 x^{10} - 0.003712111708 x^{9} + 0.003128576178 x^{8} + 0.07415371270 x^{7} - 0.0443692017 x^{6} - 0.7886688775 x^{5} + 0.3332847038 x^{4} + 3.831397601 x^{3} - 0.9999918500 x^{2} - 4.999676653 x + 2.499999995 \quad (17)$$

```
> plot([poly, Dat], x=-4..4,   style=[line,point],symbolsize=20,
  symbol=box,
    color=[blue,red], size=[.5,"golden"]);
```




```
> Dat[6];
```

$$[-0.5, 4.315138568] \quad (18)$$

```
> Dat2:=Dat;   # make a copy of the data
```

$$Dat2 := [[-3.0, 1.500718640], [-2.5, -1.333413875], [-2.0, -1.506526723], [-1.5, 0.930868789], [-1.0, 3.678347877], [-0.5, 4.315138568], [0., 2.500000000], [0.5, 0.225163738], [1.0, -0.094494713], [1.5, 2.079138715], [2.0, 4.852883103], [2.5, 5.617076061], [3.0, 3.459451646]] \quad (19)$$

```
> Dat2[6][2]:=Dat[6][2]+.2;
```

$$Dat2 := [[-3.0, 1.500718640], [-2.5, -1.333413875], [-2.0, -1.506526723], [-1.5, 0.930868789], [-1.0, 3.678347877], [-0.5, 4.515138568], [0., 2.500000000], [0.5, 0.225163738], [1.0, -0.094494713], [1.5, 2.079138715], [2.0, 4.852883103], [2.5, 5.617076061], [3.0, 3.459451646]] \quad (20)$$

Note that Dat[6] didn't change

```
> Dat[6],Dat2[6]
```

(21)

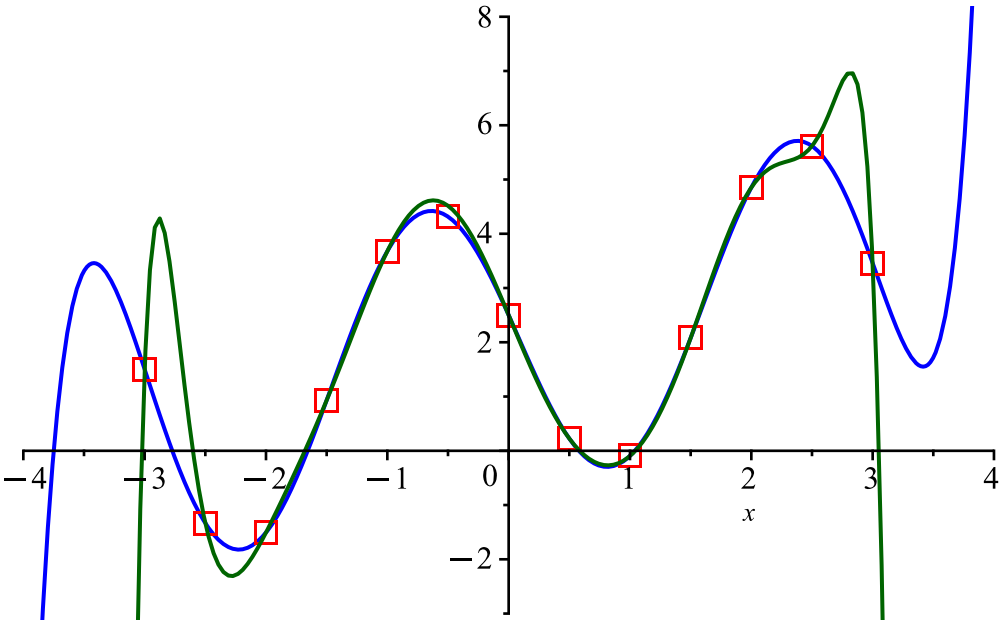
$$[-0.5, 4.315138568], [-0.5, 4.515138568] \quad (21)$$

```
> poly2:=PolynomialInterpolation(Dat2,x)
```

$$poly2 := -0.001351915462\,x^{12} + 0.000762310185\,x^{11} + 0.03034800542\,x^{10} - 0.01895019120\,x^{9} \quad (22)$$
$$- 0.2434746864\,x^{8} + 0.1974551888\,x^{7} + 0.8352077748\,x^{6} - 1.228456920\,x^{5}$$
$$- 1.014525085\,x^{4} + 4.505302043\,x^{3} - 0.314277529\,x^{2} - 5.342533747\,x + 2.499999995$$

Are these much different?

```
> plot([poly, Dat, poly2,Dat2], x=-4..4,  style=[line,point,line,
  point],symbolsize=20,symbol=box,
   color=[blue,red,"DarkGreen",black], size=[.75,"golden"]);
```




```
> sort(poly2-poly)
```

$$-0.001354497937\,x^{12} + 0.0006772479266\,x^{11} + 0.03047620272\,x^{10} - 0.01523807949\,x^{9} \quad (23)$$
$$- 0.2466032626\,x^{8} + 0.1233014761\,x^{7} + 0.8795769765\,x^{6} - 0.4397880425\,x^{5}$$
$$- 1.347809789\,x^{4} + 0.673904442\,x^{3} + 0.6857143210\,x^{2} - 0.342857094\,x$$

I want to make a side trip about saving/loading data

```
> currentdir(
  ) ;
```

"/home/campus.stonybrook.edu/ssutherland/webpage/daily" (24)

```
> currentdir("/home/campus.stonybrook.edu/ssutherland/");
```

"/home/campus.stonybrook.edu/ssutherland" (25)

```
> currentdir("/tmp");
```

"/home/campus.stonybrook.edu/ssutherland" (26)

If I use save(variable, filename) it creates filename as a maple command to assign the value of the variable.

```
> save(Dat2,"data2.txt")
```

made
Dat2 := [[-3.0, 1.500718640], [-2.5, -1.333413875], [-2.0, -1.506526723], [-1.5
, .930868789], [-1.0, 3.678347877], [-.5, 4.515138568], [0., 2.500000000], [.5,
.225163738], [1.0, -.94494713e-1], [1.5, 2.079138715], [2.0, 4.852883103], [2.5
, 5.617076061], [3.0, 3.459451646]];

```
> Dat2:="garbage";
```

$$Dat2 := \text{"garbage"} \tag{27}$$

```
> Dat2;
```

$$\text{"garbage"} \tag{28}$$

```
> read("data2.txt");
```

$$Dat2 := [[-3.0, 1.500718640], [-2.5, -1.333413875], [-2.0, -1.506526723], [-1.5, 0.930868789], [-1.0, 3.678347877], [-0.5, 4.515138568], [0., 2.500000000], [0.5, 0.225163738], [1.0, -0.094494713], [1.5, 2.079138715], [2.0, 4.852883103], [2.5, 5.617076061], [3.0, 3.459451646]] \tag{29}$$

```
> Dat2
```

$$[[-3.0, 1.500718640], [-2.5, -1.333413875], [-2.0, -1.506526723], [-1.5, 0.930868789], [-1.0, 3.678347877], [-0.5, 4.515138568], [0., 2.500000000], [0.5, 0.225163738], [1.0, -0.094494713], [1.5, 2.079138715], [2.0, 4.852883103], [2.5, 5.617076061], [3.0, 3.459451646]] \tag{30}$$

```
> save(Dat,Dat2,poly,"data2.txt")
  # save three things there.
> with(HTTP);
```

$$[Code, Form, Get, Post, URLDecode, URLEncode, URLParse] \tag{31}$$

```
> Get("https://www.math.stonybrook.edu/~scott/mat331.
  spr24/daily/extras/wiggly.txt")
```

$$\begin{aligned} &200, \text{"Dat := [} \\ &[-3.0,\ 1.500718640], \\ &[-2.5,\ -1.333413875], \\ &[-2.0,\ -1.506526723], \\ &[-1.5,\ \ .930868789], \\ &[-1.0,\ \ 3.678347877], \\ &[\ -.5,\ \ 4.315138568], \\ &[\ 0.0,\ \ 2.500000000], \\ &[\ \ .5,\ \ \ .225163738], \\ &[\ 1.0,\ \ -.094494713], \\ &[\ 1.5,\ \ 2.079138715], \\ &[\ 2.0,\ \ 4.852883103], \\ &[\ 2.5,\ \ 5.617076061], \end{aligned} \tag{32}$$

[3.0, 3.459451646]

];

", table(*case_insensitive*, ["vary" = "Accept-Encoding", "date"
= "Tue, 06 Feb 2024 18:55:38 GMT", "content-length" = "297", "etag"
= ""129-61094cda2915d"", "last-modified" = "Sun, 04 Feb 2024 21:12:31 GMT", "connection"
= "Upgrade", "accept-ranges" = "bytes", "upgrade" = "h2,h2c", "server" = "Apache",
"x-frame-options" = "SAMEORIGIN", "content-type" = "text/plain"])

```
> #
  #  Maple procedure to execute maple code stored on the web
  #
  ExecuteFromWeb:=proc(URL::string, {printfile::truefalse:=false})
   local n,m, status, webfile, headers;
   # try to get the URL
   status,webfile,headers:=HTTP[Get](URL):
   if ( HTTP[Code](status) <> "OK") then
      error(HTTP[Code](status),URL);
   fi;
   # now interpret the maple on the web page
   n:=0:
   while (n < length(webfile)) do
     m:=n;
     parse(webfile,statement,lastread='n', offset=n);
     if (printfile) then printf("%s",webfile[m+1..n]); fi;
   od:
  end:
```

Let's try it.

First, set Dat to "nope", then load the file

```
> Dat:="nope";
  Dat;
```

$$Dat := \text{"nope"}$$

$$\text{"nope"} \tag{33}$$

```
> ExecuteFromWeb("https://www.math.stonybrook.edu/~scott/mat331.
  spr24/daily/extras/wiggly2.txt",printfile);
Dat := [
[-3.0, 1.500718640],
[-2.5, -1.333413875],
[-2.0, -1.506526723],
[-1.5,  .930868789],
[-1.0, 3.678347877],
[ -.5, 4.315138568],
[ 0.0, 2.500000000],
[  .5,  .225163738],
[ 1.0, -.094494713],
[ 1.5, 2.079138715],
[ 2.0, 4.852883103],
[ 2.5, 5.617076061],
[ 3.0, 3.459451646]
```

```
] ;
```

```
> Dat[3]
```

$$[-2.0, -1.506526723] \quad (34)$$

```
> ClassURL:="https://www.math.stonybrook.edu/~scott/mat331.
  spr24/daily/extras/"
```

$$ClassURL := \text{"https://www.math.stonybrook.edu/~scott/mat331.spr24/daily/extras/"} \quad (35)$$

```
> cat(ClassURL,"wiggly2.txt")
```

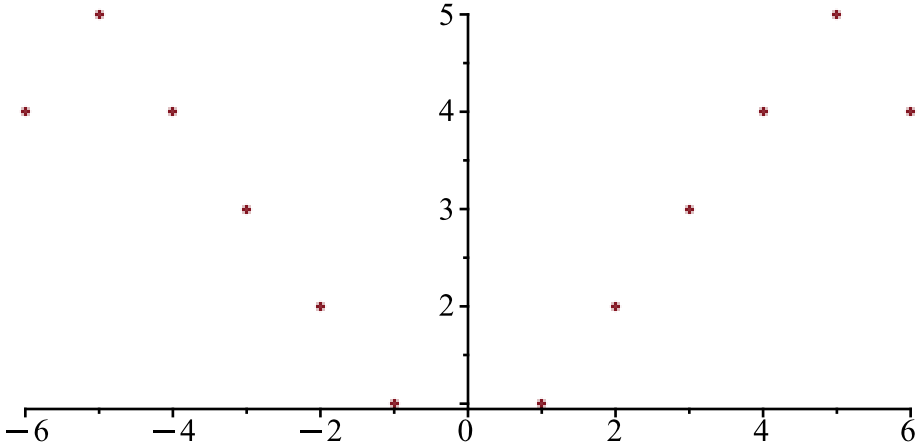
$$\text{"https://www.math.stonybrook.edu/~scott/mat331.spr24/daily/extras/wiggly2.txt"} \quad (36)$$

OK, back to math.
Let's try another.

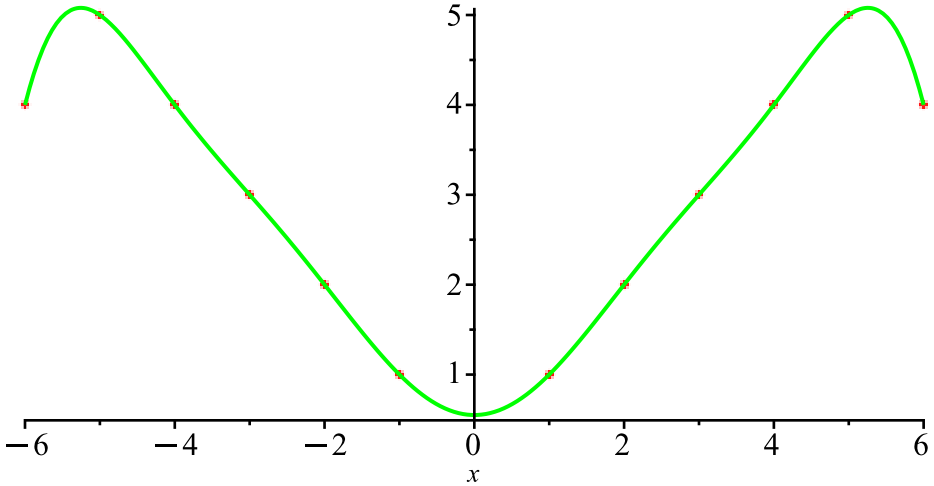
```
> bendy:=[[-6,4],seq([x,-x],x=-5..-1),seq([x,x],x=1..5),[6,4]]
```

$$bendy := [[-6, 4], [-5, 5], [-4, 4], [-3, 3], [-2, 2], [-1, 1], [1, 1], [2, 2], [3, 3], [4, 4], [5, 5], [6, 4]] \quad (37)$$

```
> plot(bendy,-6..6,style=point);
```




```
> plot([bendy,PolynomialInterpolation(bendy,x)],
    x=-6..6,style=[point,line], color=[red,green]);
```

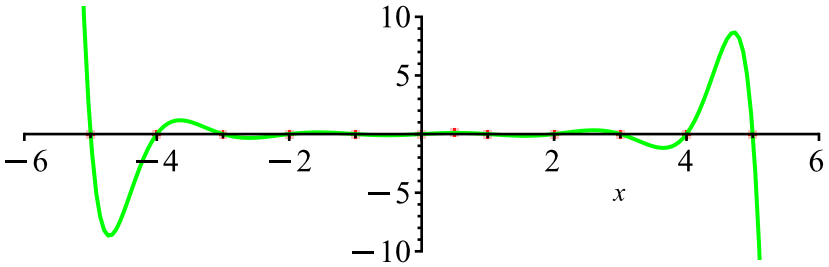



```
> flattish:=[seq([i,0],i=-5..5),[0.5,.1]];
```

$$flattish := [[-5, 0], [-4, 0], [-3, 0], [-2, 0], [-1, 0], [0, 0], [1, 0], [2, 0], [3, 0], [4, 0], \quad (38)$$

[5, 0], [0.5, 0.1]]

```
> plot([flattish,PolynomialInterpolation(flattish,x)],
    x=-6..6,style=[point,line], color=[red,green]);
```

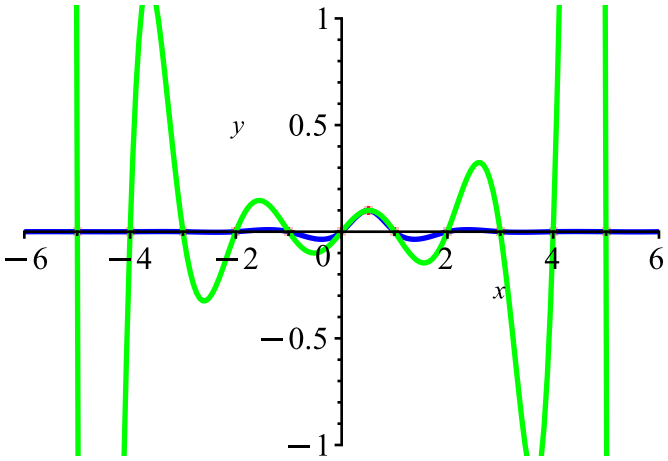



```
> mine:=Spline(flattish,x);
```

$$mine := \begin{cases} -0.00289156171106117 - 0.000578312342212234\,x + 0.000578312342212234\,(x \\ 0.00462649873769787 + 0.00115662468442447\,x + 0.00173493702663670\,(x+4)^2 - 0.0028915\text{6} \\ -0.0121445591864569 - 0.00404818639548564\,x - 0.00693974810654681\,(x+3)^2 + 0.010987\text{9} \\ 0.0300722417950362 + 0.0150361208975181\,x + 0.0260240553995505\,(x+2)^2 - 0.041060176 \\ -0.0560962971945867 - 0.0560962971945867\,x - 0.0971564734916553\,(x+1)^2 + 0.153252\text{7} \\ 0.209349067880829\,x + 0.362601838567071\,x^2 - 0.762599948657456\,x^3 \\ 0.0999995275225962 + 9.44954807569065 \times 10^{-7}\,x - 0.781298084419113\,(x-0.5)^2 + 0.762592\text{3} \\ 0.209352847700059 - 0.209352847700059\,x + 0.362590499109380\,(x-1)^2 - 0.153237651\text{4} \\ -0.112230392581475 + 0.0561151962907374\,x - 0.0971224551185840\,(x-2)^2 + 0.041007\text{25} \\ 0.0453238123886725 - 0.0151079374628908\,x + 0.0258993213649557\,(x-3)^2 - 0.010791\text{383} \\ -0.0172662142433038 + 0.00431655356082595\,x - 0.00647483034123893\,(x-4)^2 + 0.0021582\text{7} \end{cases}$$

```
> plot([mine,flattish,PolynomialInterpolation(flattish,x)],
    x=-6..6,y=-1..1,
```

```
style=[line,point,line], color=[blue,red,green],thickness=2,
size=[.5,.7]);
```



This is a cubic spline... ie, a piecewise curve that goes through the points, and is made up of cubics that fit together smoothly. We will talk about this more next time.