February 1 2023

Today is national dark chocolate day!

>

Let's talk avout factoring, relative to problem 2.

> $f(x) := x^4 - 4;$

$$f(x) := x^4 - 4 \quad (1)$$

> $factor(f(x))$

$$(x^2 - 2)(x^2 + 2) \quad (2)$$

this means factor over rationals.... but we want over reals

> $solve(f(x) = 0);$

$$\sqrt{2}, -\sqrt{2}, \mathrm{I}\sqrt{2}, -\mathrm{I}\sqrt{2} \quad (3)$$

> $factor(f(x), \mathrm{sqrt}(2))$

$$-(x + \sqrt{2})(-x + \sqrt{2})(x^2 + 2) \quad (4)$$

> $factor(f(x), \{\mathrm{sqrt}(2), I\})$

$$(x + \sqrt{2})(\mathrm{I}\sqrt{2} - x)(-x + \sqrt{2})(x + \mathrm{I}\sqrt{2}) \quad (5)$$

> $factor(x^2 + 4, I)$

$$-(-x + 2\,\mathrm{I})(x + 2\,\mathrm{I}) \quad (6)$$

Another approach: I know that

$\sqrt{2}, -\sqrt{2}, \mathrm{I}\sqrt{2}, -\mathrm{I}\sqrt{2}$ *are all roots, so I can build the factored polynomial.*

> $sols := [solve(f(x) = 0)];$

$$sols := [\sqrt{2}, -\sqrt{2}, \sqrt{2}\,\mathrm{I}, -\mathrm{I}\sqrt{2}] \quad (7)$$

added later open if you want.

> $maketerm(r) := x - r$

$$maketerm := r \mapsto x - r \quad (1.1)$$

> $maketerm(5)$

$$x - 5 \quad (1.2)$$

> $maketerm(sols[2])$

$$x + \sqrt{2} \quad (1.3)$$

> $maketerm(sols)$

$$x + [-\sqrt{2}, \sqrt{2}, -\mathrm{I}\sqrt{2}, \sqrt{2}\,\mathrm{I}] \quad (1.4)$$

> $map(maketerm, sols)$

$$[x - \sqrt{2}, x + \sqrt{2}, x - \sqrt{2}\,\mathrm{I}, x + \sqrt{2}\,\mathrm{I}] \quad (1.5)$$

But I don't really need maketerm directly, just tell it to do that this time.

> $terms := map(r \rightarrow x - r, sols);$

$$terms := [x - \sqrt{2}, x + \sqrt{2}, x - \sqrt{2}\,\mathrm{I}, x + \sqrt{2}\,\mathrm{I}] \quad (8)$$

> $?product$

```
> product(terms);
Error, (in product) wrong number or type of arguments
> product( {rabbit, dog, 7})
Error, (in product) wrong number or type of arguments
> product(x^2, x=1..3)
```
$$36 \quad (9)$$
```
> product(thing(i), i=1..3);
```
$$thing(1)\, thing(2)\, thing(3) \quad (10)$$
```
> terms[2]
```
$$x + \sqrt{2} \quad (11)$$
```
> product(terms[i], i=1..4);
```
$$\left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\,\mathrm{I}\right)\left(x + \sqrt{2}\,\mathrm{I}\right) \quad (12)$$
```
>
```
How long is *terms*?
```
> nops(terms)
```
$$4 \quad (13)$$
```
> nops([1, 18. rrr, KK, seq(i=1..10)]);
```
$$5 \quad (14)$$
```
> product(terms[i], i=1..nops(terms));
```
$$\left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\,\mathrm{I}\right)\left(x + \sqrt{2}\,\mathrm{I}\right) \quad (15)$$
```
> solve(x^5 -x^2 + 3 = 0, x);
```
$$RootOf\left(\_Z^5 - \_Z^2 + 3, index=1\right), RootOf\left(\_Z^5 - \_Z^2 + 3, index=2\right), RootOf\left(\_Z^5 - \_Z^2 + 3, index=3\right), RootOf\left(\_Z^5 - \_Z^2 + 3, index=4\right), RootOf\left(\_Z^5 - \_Z^2 + 3, index=5\right) \quad (16)$$
```
> evalf(%);
```
$$1.047313491 + 0.6082313004\,\mathrm{I}, -0.4881437136 + 1.261168584\,\mathrm{I}, -1.118339554, -0.4881437136 - 1.261168584\,\mathrm{I}, 1.047313491 - 0.6082313004\,\mathrm{I} \quad (17)$$
```
> listOfSols := [%]
```
$$listOfSols := [1.047313491 + 0.6082313004\,\mathrm{I}, -0.4881437136 + 1.261168584\,\mathrm{I}, -1.118339554, -0.4881437136 - 1.261168584\,\mathrm{I}, 1.047313491 - 0.6082313004\,\mathrm{I}] \quad (18)$$
```
> f(x) := sqrt(x + 5)
```
$$f := x \mapsto \sqrt{x+5} \quad (19)$$
```
> f(3);
```
$$2\sqrt{2} \quad (20)$$
```
> f(-10);
```
$$\mathrm{I}\sqrt{5} \quad (21)$$
```
>
```
What if I want f to not return complex numbers, just complain if x-5<0

a new line is shift-enter

```
> f:=proc(x)
    if x < -5 then
      return(x + 5);
    else
      return(sqrt(x + 5));
    fi;
  end;
```

$f := \textbf{proc}(x)\ \textbf{if}\ x < -5\ \textbf{then return}\ x + 5\ \textbf{else return}\ \mathrm{sqrt}(x+5)\ \textbf{end if end proc}$ **(22)**

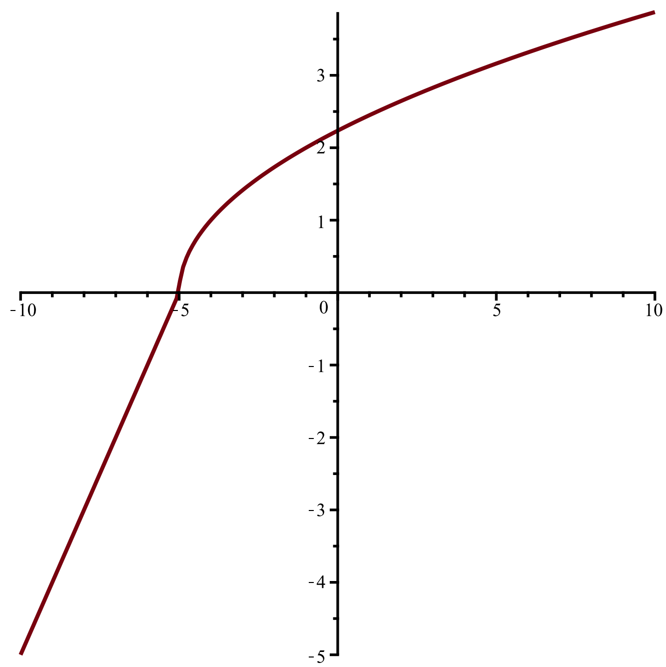
```
> f(2);
```

$$\sqrt{7}$$ **(23)**

```
> f(-10);
```

$$-5$$ **(24)**

```
> plot(f,-10..10);
```



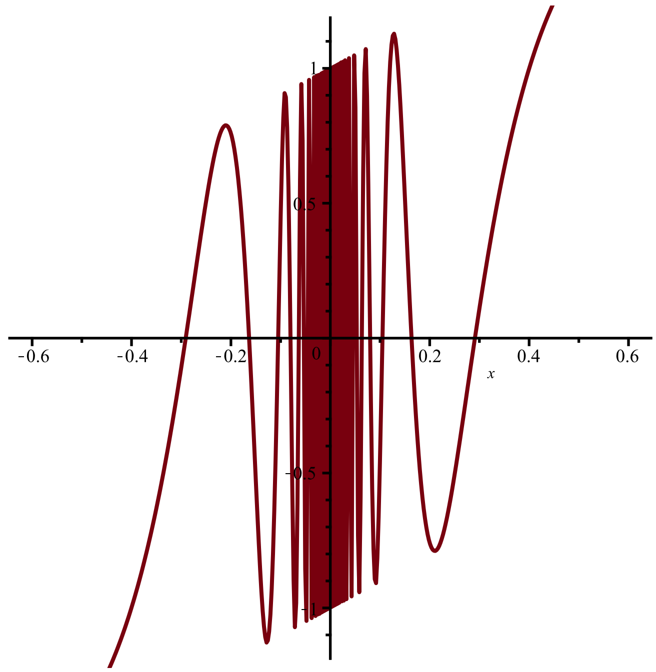

```
> f(turkey)
Error, (in f) cannot determine if this expression is true or false: turkey < -5
>
> ithprime(10);
```

$$29$$ **(25)**

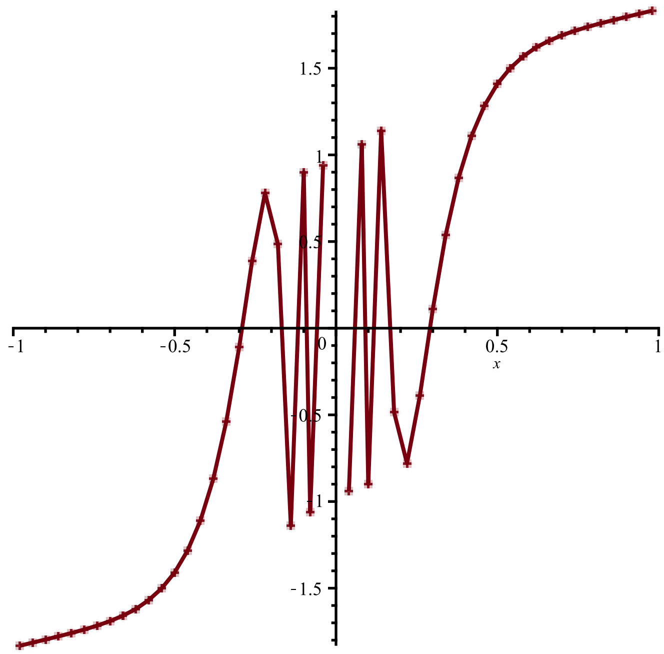
```
> ithprime(1000);
```

$$7919$$ **(26)**

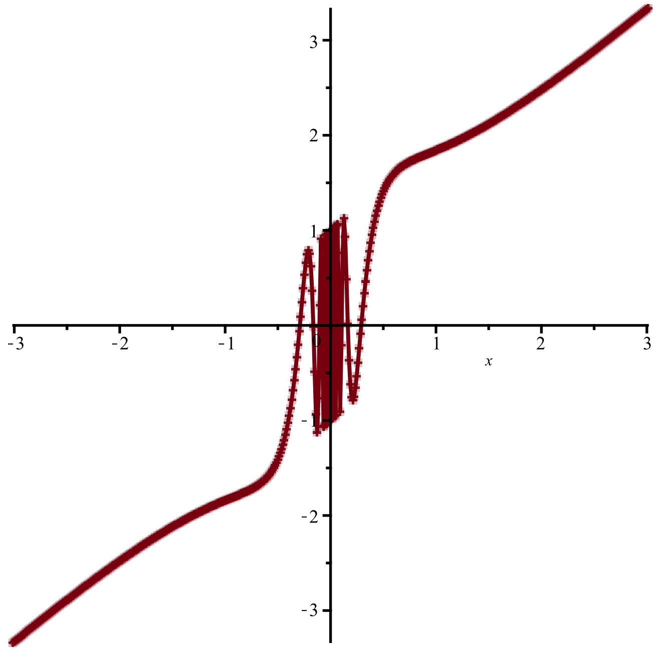
$> plot\left(\sin\left(\frac{1}{x}\right) + x, x=-1..1\right)$



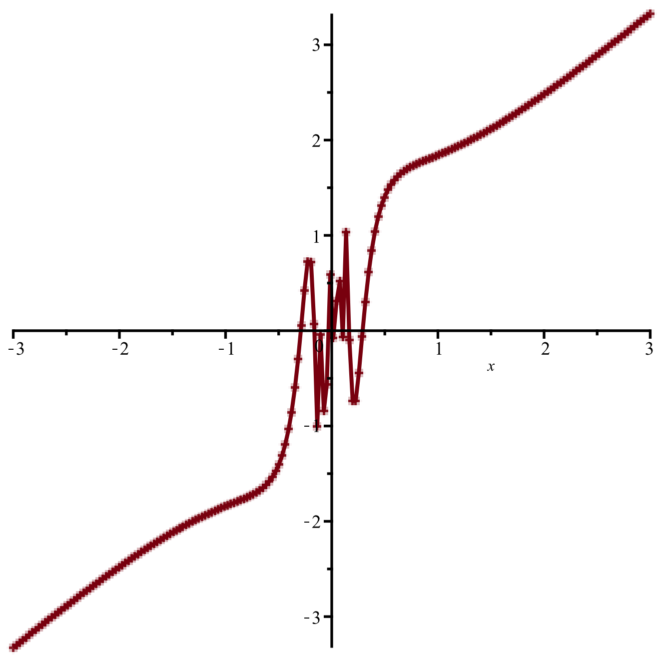
> $plot\left(\sin\left(\frac{1}{x}\right) + x, x = -1..1, numpoints = 50, style = pointline\right)$



> $plot\left(\sin\left(\frac{1}{x}\right) + x, x = -3..3, style = pointline\right)$



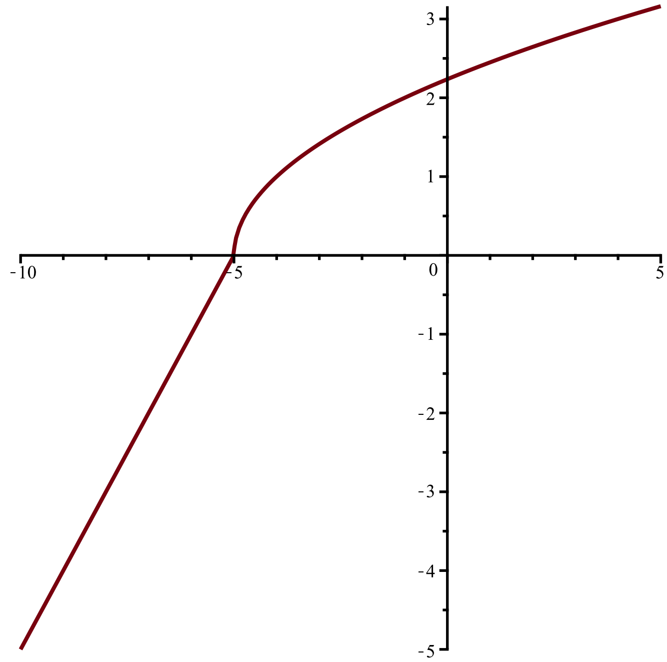
> $plot\left(\sin\left(\frac{1}{x}\right) + x, x = -3 ..3, style = pointline, adaptive = false\right)$



> $plot(f(x), x = -10 ..5);$

Error, (in f) cannot determine if this expression is true or false: x < -5

> $plot(f, -10 ..5)$



>

What???/?

> $f(x)$

Error, (in f) cannot determine if this expression is true or false: x < -5

> $f(turkey)$

Error, (in f) cannot determine if this expression is true or false: turkey < -5

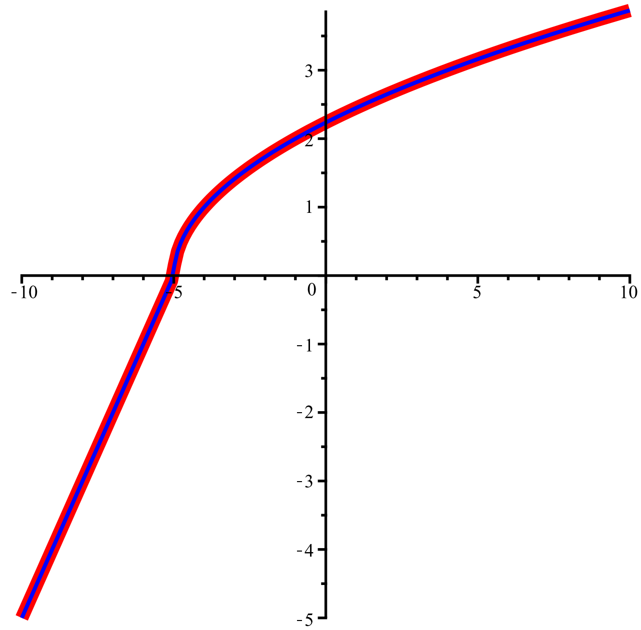
Trouble is that maple is trying to evaluate f(x) "too early", before x has a value.

> ?*piecewise*

> $g := x \to piecewise(x > -5, \mathrm{sqrt}(x+5), x+5);$

$$g := x \mapsto \begin{cases} \sqrt{x+5} & -5 < x \\ x+5 & otherwise \end{cases} \tag{27}$$

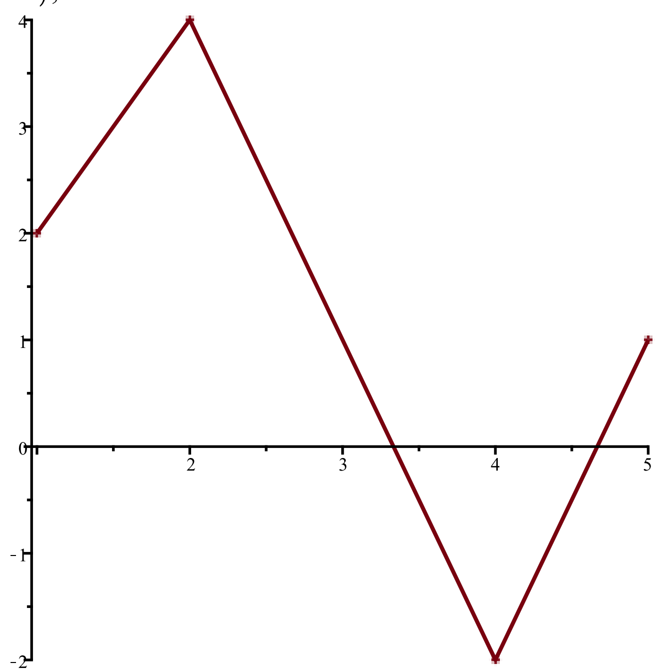
> $plot([g, f], -10..10, thickness = [5, 1], color = [red, blue]);$



> $pts := [\ [1,2], [2,4], [4,-2], [5,1]\ ];$

$$pts := [[1,2],[2,4],[4,-2],[5,1]] \quad \textbf{(28)}$$

> $plot(pts, style=pointline);$



How do I find the unique cubic that goes through those 4 points?

Next time.