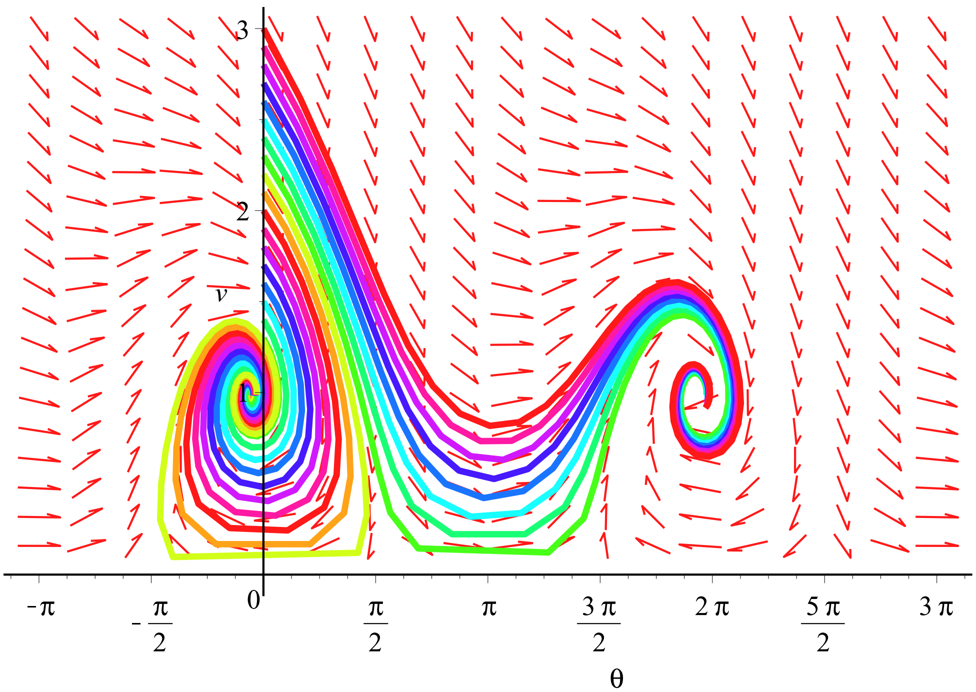
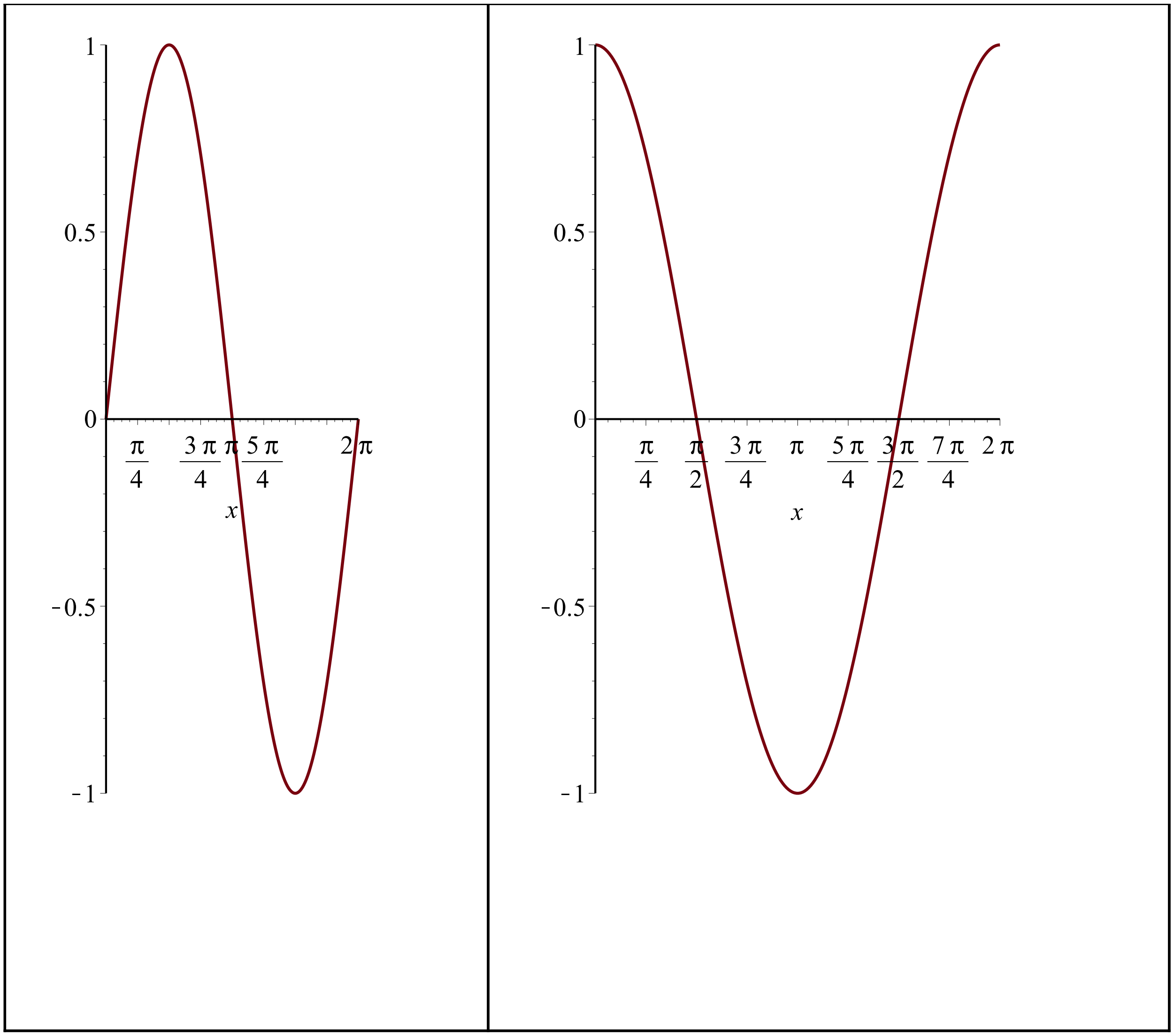
```
>
> with(DEtools):
> xphug:=R-> [diff(theta(t),t)=v(t)-cos(theta(t))/v(t),
             diff(v(t),t)    = -sin(theta(t)) - R*(v(t))^2,
             diff(x(t),t)    =v(t)*cos(theta(t)),
             diff(y(t),t)    =v(t)*sin(theta(t))]:
  phug:=R->[xphug(R)[1],xphug(R)[2]]:
> DEplot(phug(0.2), [theta, v], t = 0 .. 10, theta = -Pi .. 3*Pi, v
  = 0 .. 3, [seq([theta(0) = 0, v(0) = vel], vel = 1 .. 3, .1)],
  tickmarks = [piticks, default], linecolor = [seq(COLOR(HUE, h),h=
  1..3, .1)]);
```



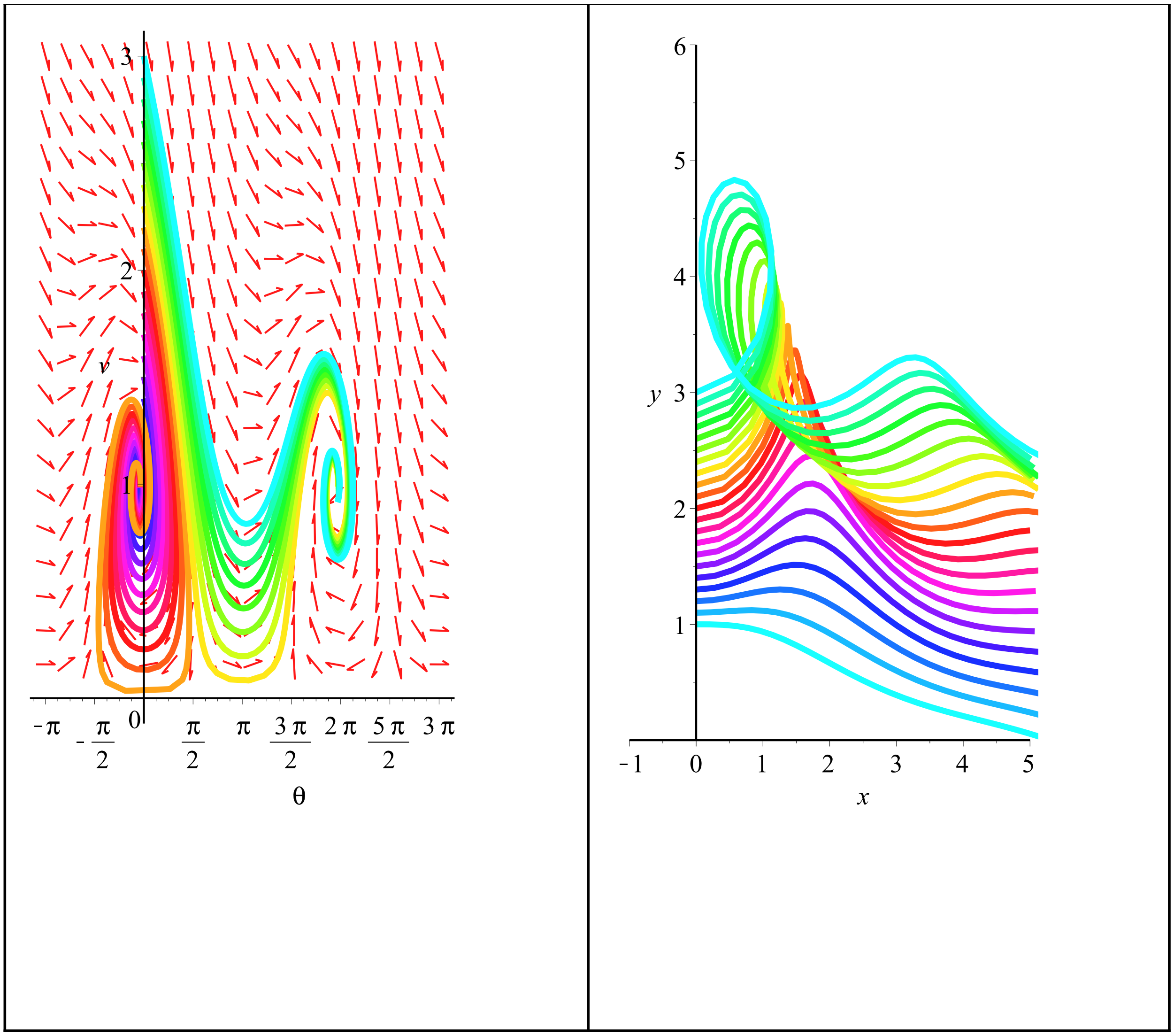
> *with*(*plots*) :

>

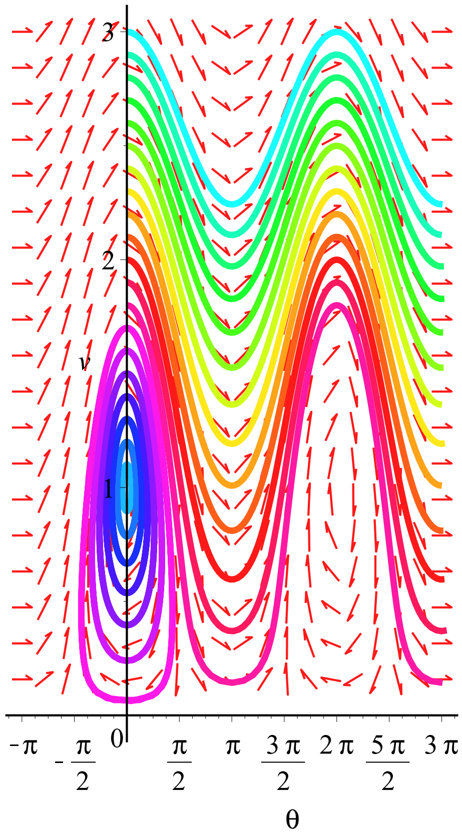
> *display*(*Matrix*([[*plot*(sin(*x*), *x* = 0 ..2·Pi), *plot*(cos(*x*), *x* = 0 ..2·Pi)]]));

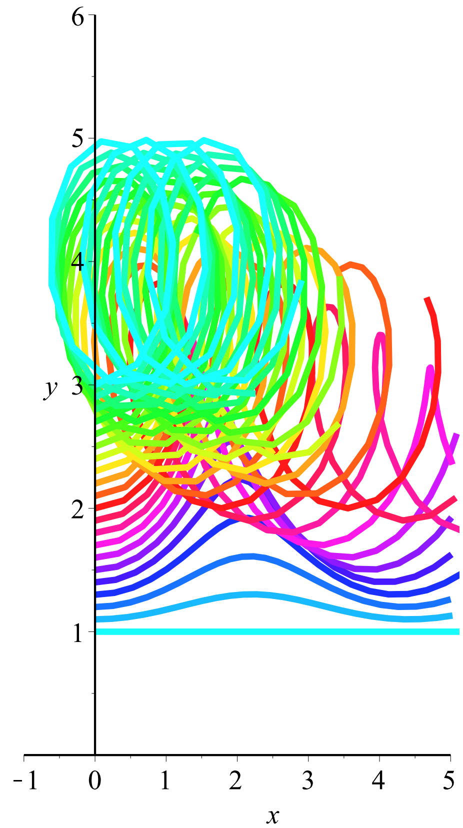



```
> display(Matrix([[
  # left pic
    DEplot(phug(0.2), [theta, v], t = 0 .. 10,
    theta = -Pi .. 3*Pi, v = 0 .. 3,
    [seq([theta(0) = 0, v(0) = vel], vel = 1 .. 3, .1)], tickmarks
  = [piticks, default], numpoints=200,
  linecolor = [seq(COLOR(HUE, h/2),h=1..3, .1)]),
  # right pic
  DEplot(xphug(0.2), [theta, v,x,y],
     t = 0 .. 10, theta = -Pi .. 3*Pi, v = 0 .. 3,
     x=-1..5, y=0..6,
    [seq([theta(0)=0, v(0)=vel, x(0)=0, y(0)=vel],
     vel = 1 .. 3, .1)], scene=[x,y],
    linecolor = [seq(COLOR(HUE, h/2),h=1..3, .1)])
  ]]));
```

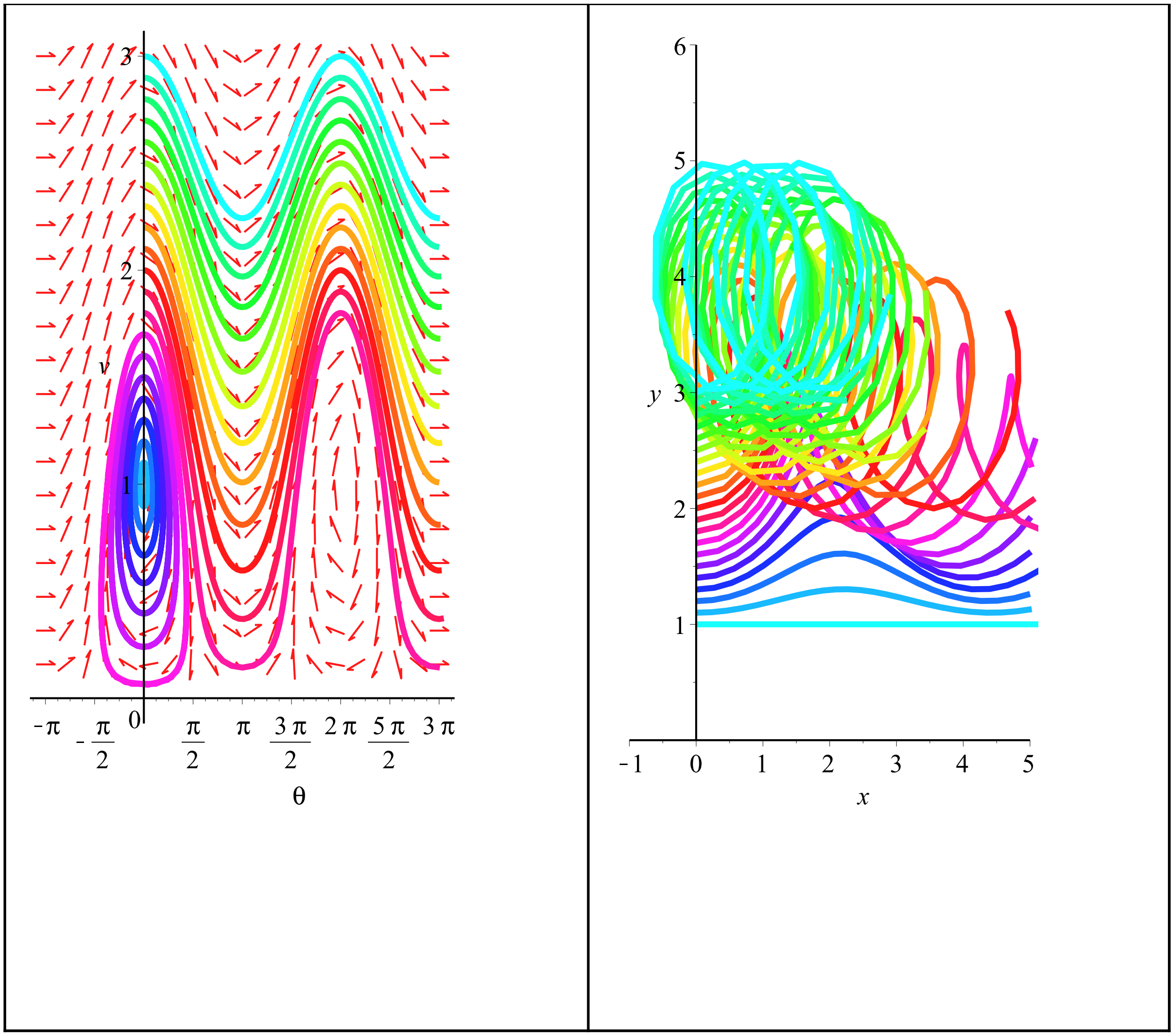



```
> gliderPic:=R->display(Matrix([[
  # left pic
    DEplot(phug(R), [theta, v], t = 0 .. 10,
    theta = -Pi .. 3*Pi, v = 0 .. 3,
    [seq([theta(0) = 0, v(0) = vel], vel = 1 .. 3, .1)], tickmarks
  = [piticks, default], numpoints=200,
  linecolor = [seq(COLOR(HUE, h/2),h=1..3, .1)]),
  # right pic
  DEplot(xphug(R), [theta, v,x,y],
     t = 0 .. 10, theta = -Pi .. 3*Pi, v = 0 .. 3,
     x=-1..5, y=0..6,
    [seq([theta(0)=0, v(0)=vel, x(0)=0, y(0)=vel],
     vel = 1 .. 3, .1)], scene=[x,y],
    linecolor = [seq(COLOR(HUE, h/2),h=1..3, .1)])
  ]])):
> gliderPic(0);
```






```
> glidermovie:= [seq(gliderPic(R), R=0..4, .1)]:
> display(glidermovie, insequence = true);
```



> $solve\left(\left\{v - \frac{\cos(\text{theta})}{v} = 0,\ -\sin(\text{theta}) - R\cdot v^2 = 0\right\}, \{\text{theta}, v\}\right);$

$$\{\theta = \arctan(-RootOf(-1 + (R^2+1)\ \_Z^2)\ R, RootOf(-1 + (R^2+1)\ \_Z^2)), v = RootOf(\_Z^2 - RootOf(-1 + (R^2+1)\ \_Z^2))\} \quad \textbf{(1)}$$

> $convert(\%, radical);$

$$\left\{\theta = \arctan\left(-\sqrt{\frac{1}{R^2+1}}\ R, \sqrt{\frac{1}{R^2+1}}\right), v = \left(\frac{1}{R^2+1}\right)^{1/4}\right\} \quad \textbf{(2)}$$

> $assume(R > 0);$

> $solve\left(\left\{v - \frac{\cos(\text{theta})}{v} = 0,\ -\sin(\text{theta}) - R\cdot v^2 = 0\right\}, \{\text{theta}, v\}\right) : convert(\%, radical);$

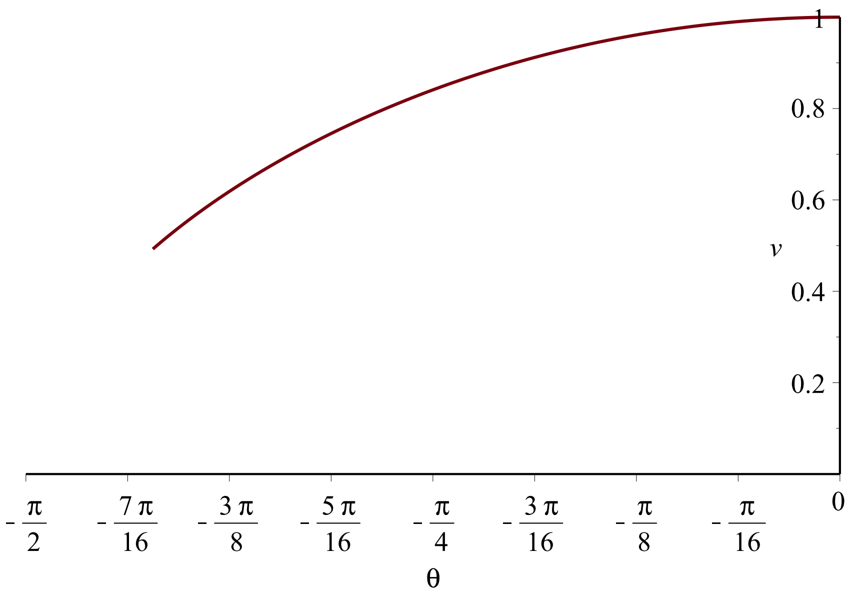
$$\left\{\theta = -\arctan(R\sim), v = \left(\frac{1}{R\sim^2+1}\right)^{1/4}\right\} \quad \textbf{(3)}$$

> $Fix := R \rightarrow \left[-\arctan(R), \left(\frac{1}{1+R^2}\right)^{\frac{1}{4}}\right];$

(4)

$$Fix := R \rightarrow \left[-\arctan(R), \left(\frac{1}{1+R^2} \right)^{1/4} \right] \qquad \textbf{(4)}$$

> $plot\left([op(Fix(R)), R = 0..4], \mathrm{theta} = -\frac{\mathrm{Pi}}{2}..0, v = 0..1 \right);$



> $phug(R);$

$$\left[\frac{d}{dt}\, \theta(t) = v(t) - \frac{\cos(\theta(t))}{v(t)}, \frac{d}{dt}\, v(t) = -\sin(\theta(t)) - R\sim v(t)^2 \right] \qquad \textbf{(5)}$$

> $map(e \rightarrow rhs(e), phug(R));$

$$\left[v(t) - \frac{\cos(\theta(t))}{v(t)}, -\sin(\theta(t)) - R\sim v(t)^2 \right] \qquad \textbf{(6)}$$

> $subs(\{v(t) = v, \mathrm{theta}(t) = \mathrm{theta}\}, \%);$

$$\left[v - \frac{\cos(\theta)}{v}, -\sin(\theta) - R\sim v^2 \right] \qquad \textbf{(7)}$$

> $F := unapply(\%, (\mathrm{theta}, v, R));$

$$F := (\theta, v, R\sim) \rightarrow \left[v - \frac{\cos(\theta)}{v}, -\sin(\theta) - R\sim v^2 \right] \qquad \textbf{(8)}$$

> $F := unapply(subs(\{v(t) = v, \mathrm{theta}(t) = \mathrm{theta}\}, map(e \rightarrow rhs(e), phug(R))), (\mathrm{theta}, v, R));$

$$F := (\theta, v, R\sim) \rightarrow \left[v - \frac{\cos(\theta)}{v}, -\sin(\theta) - R\sim v^2 \right] \qquad \textbf{(9)}$$

> $F(\mathrm{theta}, v, .2);$

$$\left[v - \frac{\cos(\theta)}{v}, -\sin(\theta) - 0.2\, v^2 \right] \qquad \textbf{(10)}$$

> ?*Jacobian*

> *with*(*VectorCalculus*) :

> *Jacobian*(*F*(theta, *v*, *R*), [theta, *v*] = *Fix*(*R*));

$$\begin{bmatrix} -\dfrac{R\sim}{\sqrt{R\sim^2+1}\left(\dfrac{1}{R\sim^2+1}\right)^{1/4}} & 1+\dfrac{1}{\sqrt{R\sim^2+1}\sqrt{\dfrac{1}{R\sim^2+1}}} \\ -\dfrac{1}{\sqrt{R\sim^2+1}} & -2\,R\sim\left(\dfrac{1}{R\sim^2+1}\right)^{1/4} \end{bmatrix} \quad \textbf{(11)}$$

> *simplify*(%);

$$\begin{bmatrix} -\dfrac{R\sim}{(R\sim^2+1)^{1/4}} & 2 \\ -\dfrac{1}{\sqrt{R\sim^2+1}} & -\dfrac{2\,R\sim}{(R\sim^2+1)^{1/4}} \end{bmatrix} \quad \textbf{(12)}$$

> *Jack* := *simplify*(*Jacobian*(*F*(theta, *v*, *R*), [theta, *v*] = *Fix*(*R*)));

$$Jack := \begin{bmatrix} -\dfrac{R\sim}{(R\sim^2+1)^{1/4}} & 2 \\ -\dfrac{1}{\sqrt{R\sim^2+1}} & -\dfrac{2\,R\sim}{(R\sim^2+1)^{1/4}} \end{bmatrix} \quad \textbf{(13)}$$

> with(LinearAlgebra) :
Trace(*Jack*);

$$-\frac{3\,R\sim}{(R\sim^2+1)^{1/4}} \quad \textbf{(14)}$$

> *Determinant*(*Jack*);

$$2\sqrt{R\sim^2+1} \quad \textbf{(15)}$$

> *Eigenvalues*(*Jack*);

$$\begin{bmatrix} \dfrac{1}{2}\,\dfrac{-3\,R\sim+\sqrt{R\sim^2-8}}{(R\sim^2+1)^{1/4}} \\ -\dfrac{1}{2}\,\dfrac{3\,R\sim+\sqrt{R\sim^2-8}}{(R\sim^2+1)^{1/4}} \end{bmatrix} \quad \textbf{(16)}$$

> *simplify*(%);

$$\begin{bmatrix} \dfrac{1}{2}\,\dfrac{-3\,R\sim+\sqrt{R\sim^2-8}}{(R\sim^2+1)^{1/4}} \\ -\dfrac{1}{2}\,\dfrac{3\,R\sim+\sqrt{R\sim^2-8}}{(R\sim^2+1)^{1/4}} \end{bmatrix} \quad \textbf{(17)}$$

>

For R>sqrt(8), have REAL eigenvalues, for 0<R<sqrt(8) complex evals.