```
>
>
```

Somehow, I neglected to save my worksheet from 4/24. Here is my best guess at a recreation of what we did.

From multivariable calculus, when I have a function of two variables like

$F(x, y) = \langle f(x, y), g(x, y) \rangle$

Taylor's theorem tells us that

$$F(a+\epsilon, b+\delta) = F(a, b) + \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} \begin{bmatrix} \epsilon \\ \delta \end{bmatrix} + O(\epsilon^2, \delta^2, \epsilon\delta)$$

(The matrix of derivatives is the Jacobian, and should be evaluated at (a, b).)

If $F(a, b) = \langle 0, 0 \rangle$, then this says that $F(a+\epsilon, b+\delta)$ is approximately equal to a matrix times the vector $\langle \epsilon, \delta \rangle$

So here, we need to look at the Jacobian.

```
> phugfunc:=unapply(phug,(theta,v));
```

$$phugfunc := (\theta, v) \to \left[\frac{v^2 - \cos(\theta)}{v}, \sin(\theta) - R v^2 \right] \quad \textbf{(1)}$$

```
> J:=VectorCalculus[Jacobian](phugfunc(theta,v),[theta,v]);
```

$$J := \begin{bmatrix} \frac{\sin(\theta)}{v} & 2 - \frac{v^2 - \cos(\theta)}{v^2} \\ \cos(\theta) & -2 R v \end{bmatrix} \quad \textbf{(2)}$$

can examine this matrix near fixed points.

```
> Fix(0);
```

$$[0, 1] \quad \textbf{(3)}$$

```
> eval(J,{theta=0,v=1,R=0});
```

$$\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \quad \textbf{(4)}$$

```
> Fix(2);evalf(%);
```

$$\left[\arctan\left(\frac{2}{5} \sqrt{5} \right), \frac{1}{5} 5^{3/4} \right]$$
$$[0.7297276561, 0.6687403050] \quad \textbf{(5)}$$

```
> eval(J,{theta=0.7297,v=0.668,R=2});
```

$$\begin{bmatrix} 0.9979731328 & 2.670404168 \\ 0.7453744297 & -2.672 \end{bmatrix} \quad \textbf{(6)}$$

Claim it is worth understanding

$$\left(\frac{dx}{dt}, \frac{dy}{dt}\right) = A\,(x, y)$$

for matrix A.

```
> A:=< < 2,0> | <0,-3> >;
```

$$A := \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix} \qquad (7)$$

What is solution of DE (dx/dt, dy/dt) = A(x,y)?

Simpler: solve dx/dt = 2x

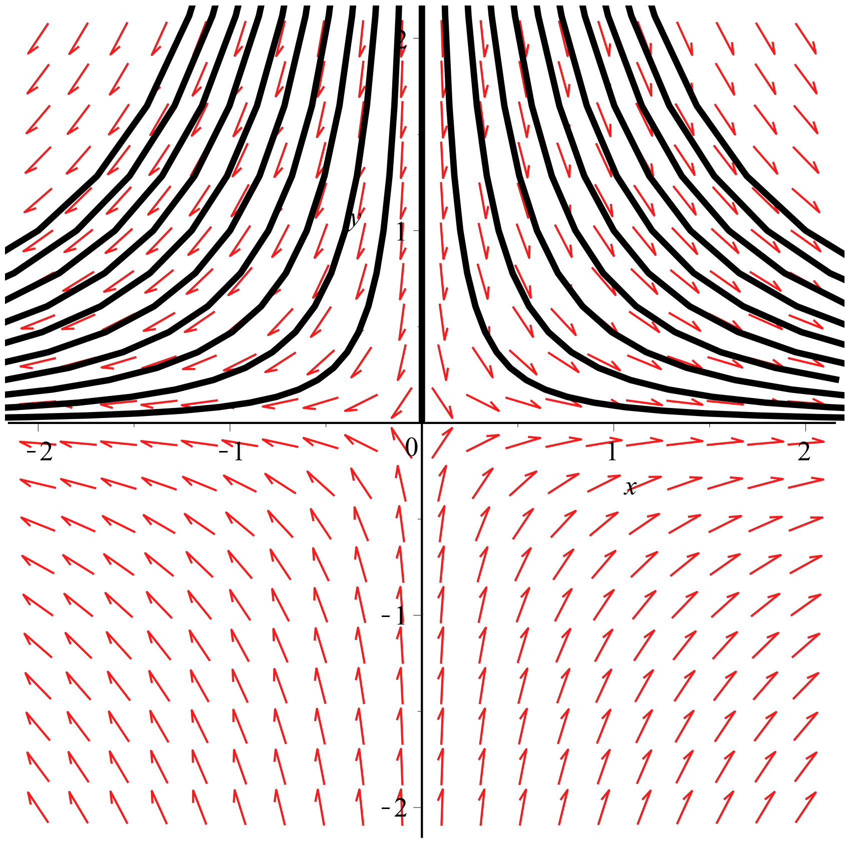
```
> dsolve(D(x)(t) = 2*x(t));
```

$$x(t) = \_C1\,e^{2\,t} \qquad (8)$$

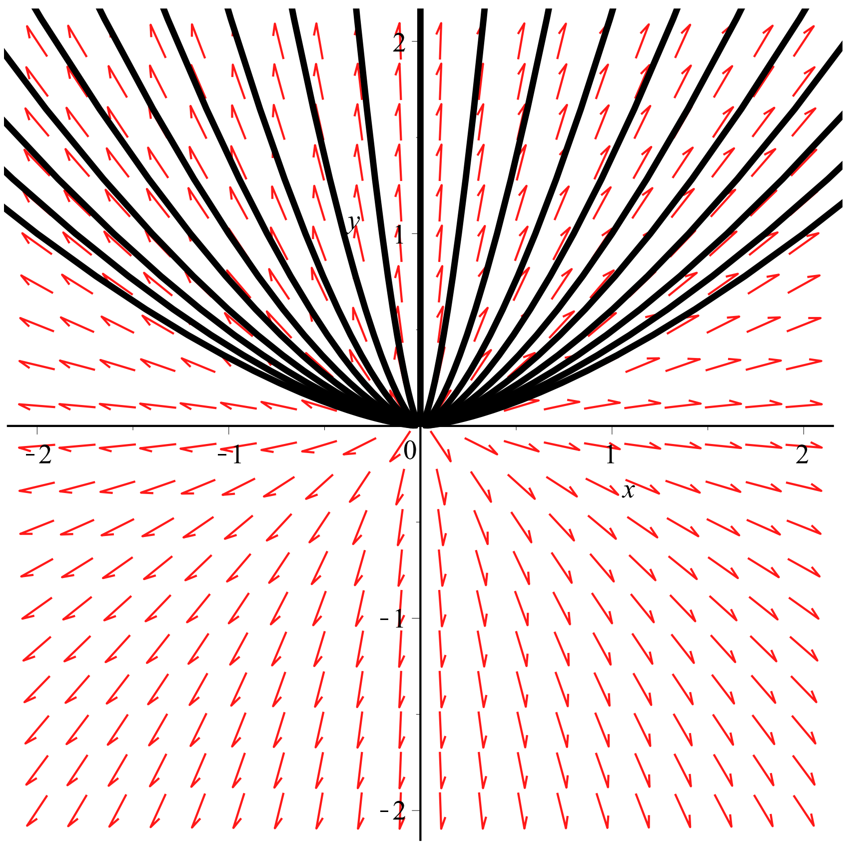
```
> < D(x)(t), D(y)(t)> = A . <x(t), y(t)>;
```

$$\begin{bmatrix} D(x)(t) \\ D(y)(t) \end{bmatrix} = \begin{bmatrix} 2\,x(t) \\ -3\,y(t) \end{bmatrix} \qquad (9)$$

```
> DEplot( [ D(x)(t) = 2*x(t), D(y)(t) = -3*y(t)],
    [x(t),y(t)], t=-2..2,
     [seq( [x(0)=i, y(0)=1],i=-2..2, .2)],
     x=-2..2, y=-2..2, linecolor=black, obsrange=false);
```



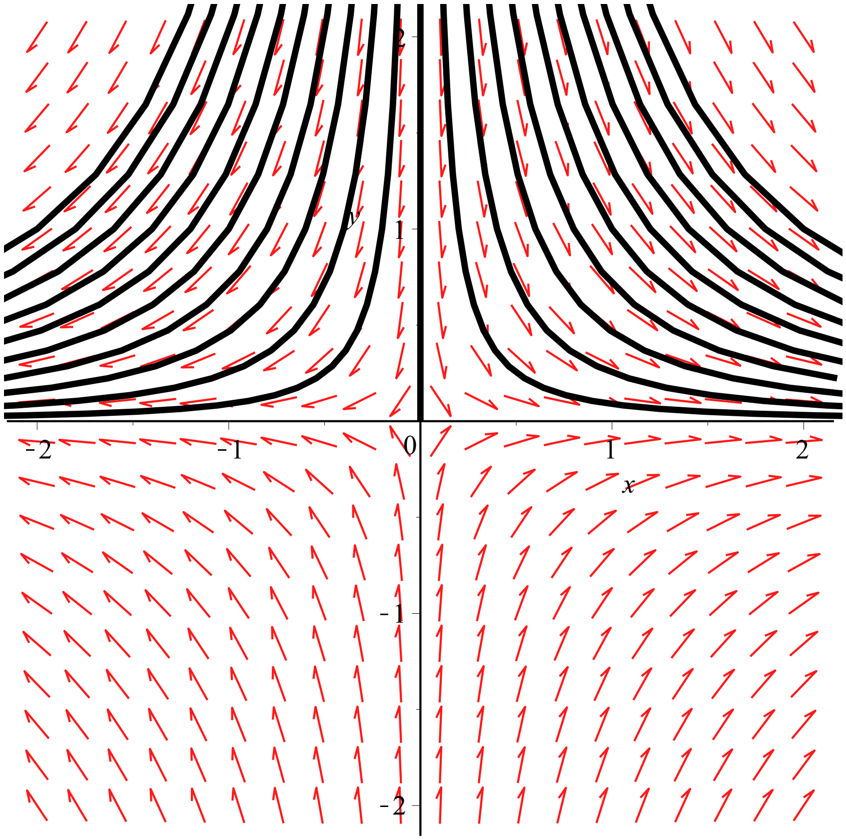

```
> DEplot( [ D(x)(t) = 2*x(t), D(y)(t) = 3*y(t)],
    [x(t),y(t)], t=-2..2,
     [seq( [x(0)=i, y(0)=1],i=-2..2, .2)],
     x=-2..2, y=-2..2, linecolor=black, obsrange=false);
```



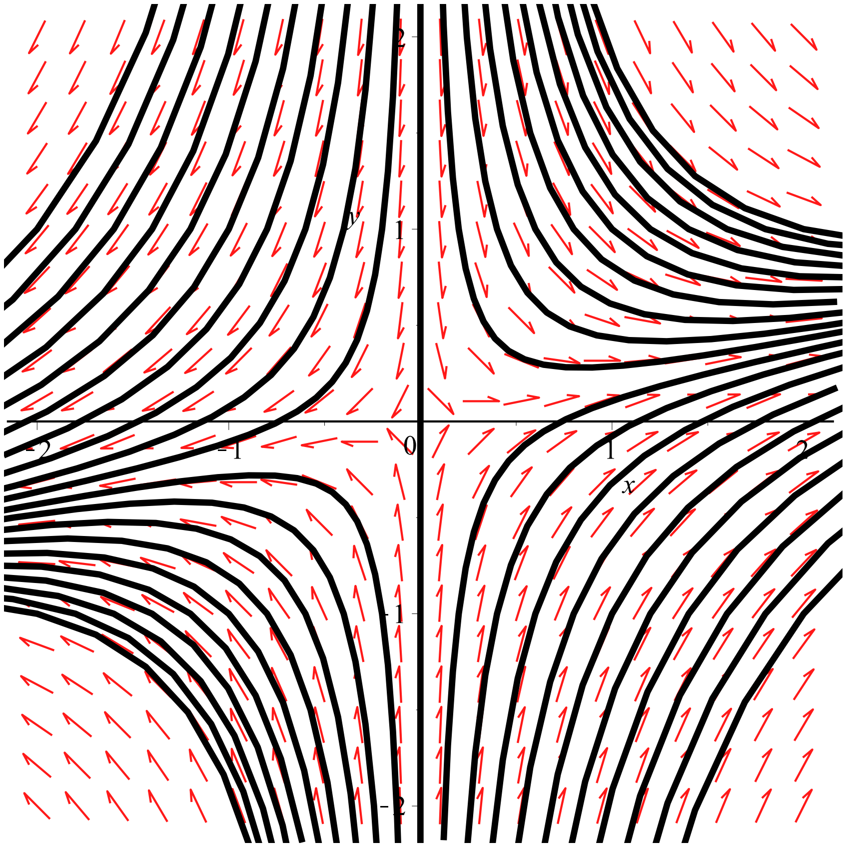

```
> MatToSys:=proc(A)
    local stuff;
    stuff:=A.<x(t),y(t)>;
    return( [ D(x)(t) = stuff[1], D(y)(t) = stuff[2]]);
  end:
> MatToSys( <<1,2>|<3,4>>);
```

$$[D(x)(t) = x(t) + 3\,y(t),\ D(y)(t) = 2\,x(t) + 4\,y(t)] \tag{10}$$

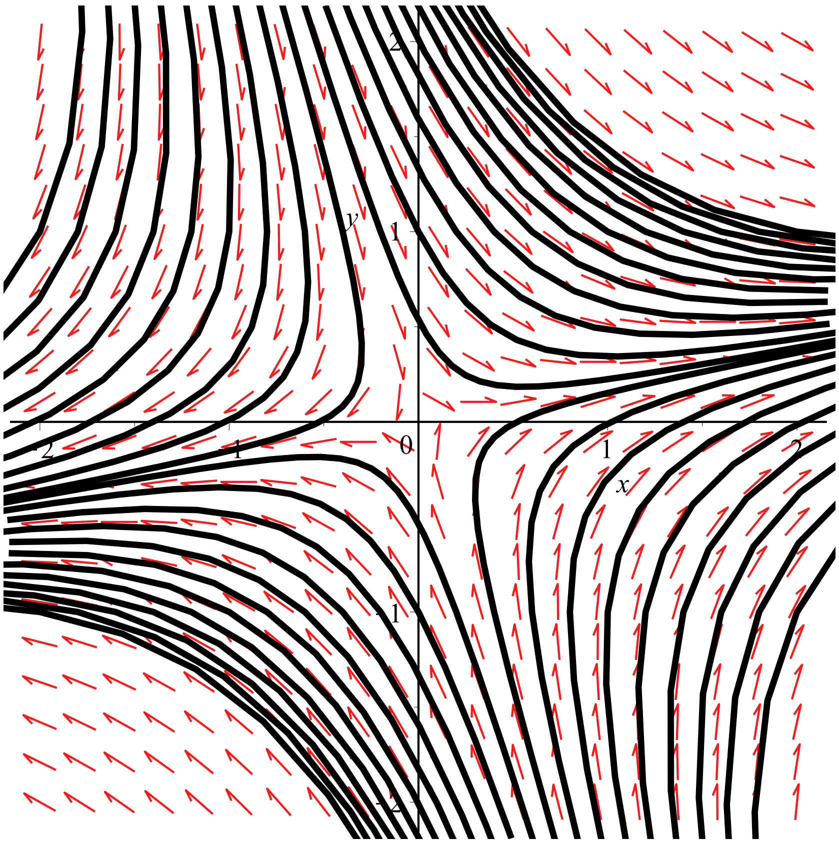
```
> DEplot( MatToSys(<<2,0>|<0,-3>>),
    [x(t),y(t)], t=-2..2,
     [seq( [x(0)=i, y(0)=1],i=-2..2, .2)],
     x=-2..2, y=-2..2, linecolor=black, obsrange=false);
```




```
> DEplot( MatToSys(<<2,1>|<0,-3>>),
    [x(t),y(t)], t=-2..2,
     [seq( [x(0)=i, y(0)=1],i=-2..2, .2),
      seq( [x(0)=i, y(0)=-1],i=-2..2, .2)],
     x=-2..2, y=-2..2, linecolor=black, obsrange=false);
```




```
> DEplot( MatToSys(<<2,1>|<1.6,-3>>),
    [x(t),y(t)], t=-2..2,
    [seq( [x(0)=i, y(0)=1],i=-2..2, .2),
     seq( [x(0)=i, y(0)=-1],i=-2..2, .2)],
    x=-2..2, y=-2..2, linecolor=black, obsrange=false);
```




```
> A:=<<2,1>|<16/10,-3>>
```

Warning, inserted missing semicolon at end of statement

$$A := \begin{bmatrix} 2 & \frac{8}{5} \\ 1 & -3 \end{bmatrix} \quad (11)$$

```
> with(LinearAlgebra):
> evalf(Eigenvectors(A));
```

$$\begin{bmatrix} 2.301785145 \\ -3.301785145 \end{bmatrix}, \begin{bmatrix} 5.301785149 & -0.3017851453 \\ 1. & 1. \end{bmatrix} \quad (12)$$

```
> JordanForm(A);
```

$$\begin{bmatrix} -\frac{1}{2} - \frac{1}{10}\sqrt{785} & 0 \\ 0 & -\frac{1}{2} + \frac{1}{10}\sqrt{785} \end{bmatrix} \quad (13)$$

```
> DEplot( MatToSys(<<0,2>|<-1,0>>),
     [x(t),y(t)], t=-2..2,
      [seq( [x(0)=i, y(0)=1],i=-2..2, .2),
```

```
seq( [x(0)=i, y(0)=-1],i=-2..2, .2)],
x=-2..2, y=-2..2, linecolor=black, obsrange=false);
```