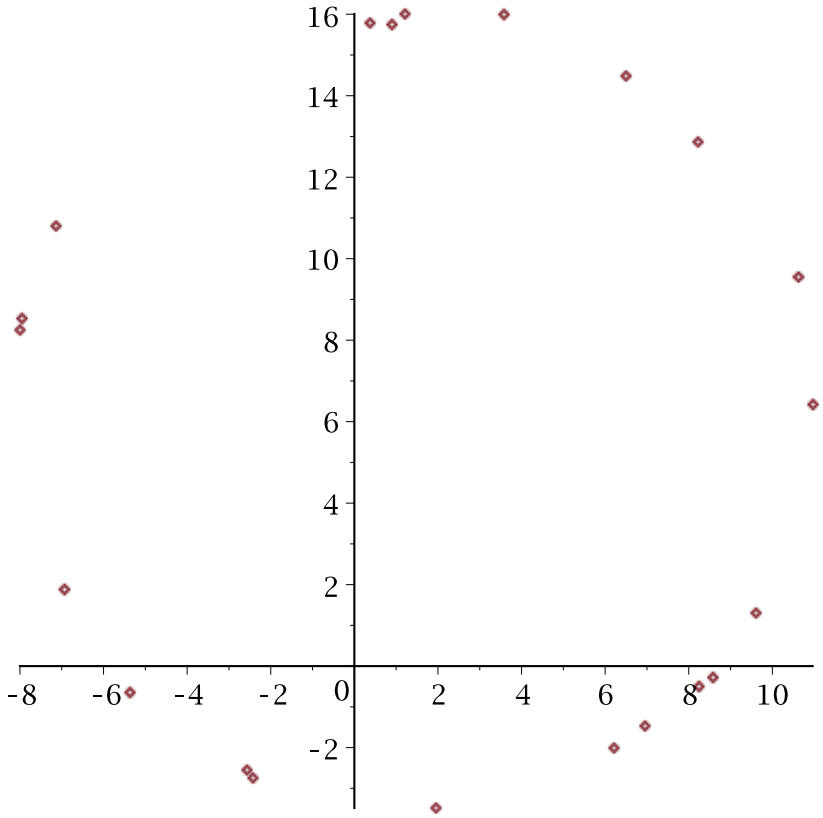
Load some useful routine here

```
> ReadFromWeb("www.math.sunysb.edu/~scott/mat331.
      spr14/problems/lsq_data.txt");
```

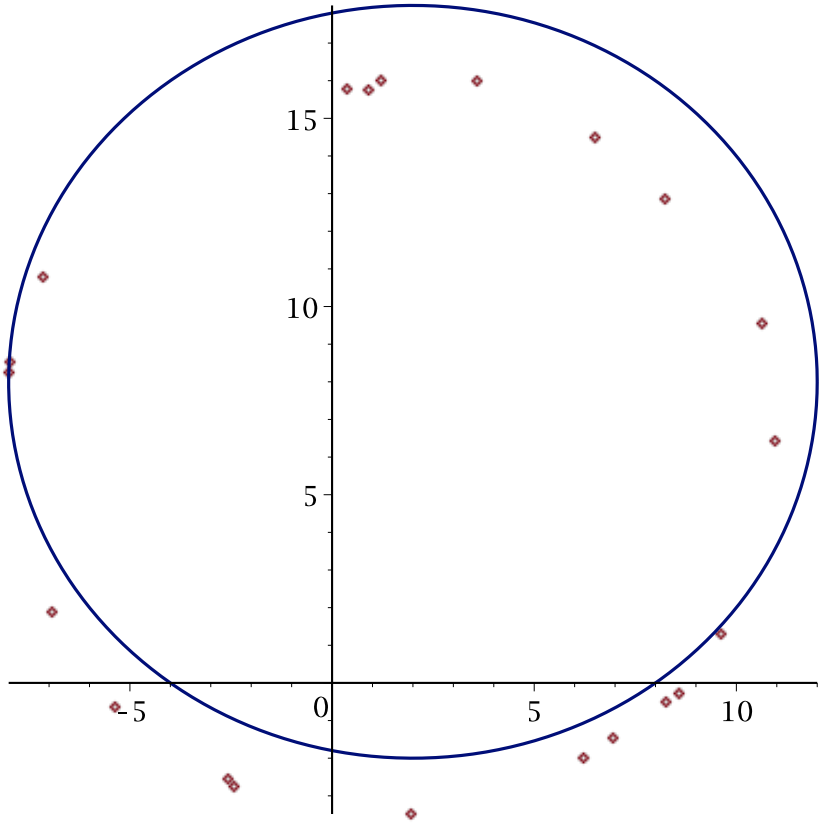
defined set_seed(s), line_pts(), bad_line_pts(), quadratic_pts(), exp_pts(), cubic_pts(), and circle_pts()

```
> data := circle_pts( ) : plot(data, style = point);
```



Guess center is (2,8), radius is 10

```
> plot([data, [10·cos(t) + 2, 10·sin(t) + 8, t = 0..2·Pi]], style = [point, line]);
```



v measures the "distance" from a point p=[x,y] to a circle of center (a,b), radius r.

```
> v := (a, b, r, p) → (p[1] − a)^2 + (p[2] − b)^2 − r^2;
```

$$v := (a, b, r, p) \rightarrow (p_1 - a)^2 + (p_2 - b)^2 - r^2 \quad (1)$$

```
> v(0, 0, sqrt(5), [1, 2]);
```

$$0 \quad (2)$$

Want to measure the "distance" from a pile of points to a target circle.

```
> H := (a, b, r, pts) → sum(v(a, b, r, pts[i])^2, i = 1 ..nops(pts)) / nops(pts);
```

$$H := (a, b, r, pts) \rightarrow \frac{\sum_{i=1}^{nops(pts)} v(a, b, r, pts_i)^2}{nops(pts)} \quad (3)$$

```
> H(2, 8, 10, data);
```

$$677.2945381 \quad (4)$$

```
> H(1.5, 8, 10, data);
```

$$625.9789486 \quad (5)$$

```
> diff(H(a, b, r, data), a);
```

$$\frac{2}{21}\left((6.954144222-a)^2+(-1.471513766-b)^2-r^2\right)(-13.90828844+2a)+\frac{2}{21}\left((6.497455235-a)^2+(14.48868588-b)^2-r^2\right)(-12.99491047+2a)+\frac{2}{21}\left((0.368707320-a)^2+(15.78153752-b)^2-r^2\right)(-0.737414640+2a)+\frac{2}{21}\left((-6.936817732-a)^2+(1.880784796-b)^2-r^2\right)(13.87363546+2a)+\frac{2}{21}\left((-7.142671381-a)^2+(10.79362146-b)^2-r^2\right)(14.28534276+2a)+\frac{2}{21}\left((10.96370257-a)^2+(6.423487122-b)^2-r^2\right)(-21.92740514+2a)+\frac{2}{21}\left((3.577460537-a)^2+(15.99249155-b)^2-r^2\right)(-7.154921074+2a)+\frac{2}{21}\left((-7.999265666-a)^2+(8.247369599-b)^2-r^2\right)(15.99853133+2a)+\frac{2}{21}\left((-2.427782838-a)^2+(-2.754320171-b)^2-r^2\right)(4.855565676+2a)+\frac{2}{21}\left((-7.958364099-a)^2+(8.533898956-b)^2-r^2\right)(15.91672820+2a)+\frac{2}{21}\left((1.205043711-a)^2+(16.00684526-b)^2-r^2\right)(-2.410087422+2a)+\frac{2}{21}\left((10.62510113-a)^2+(9.551555223-b)^2-r^2\right)(-21.25020226+2a)+\frac{2}{21}\left((8.225496557-a)^2+(12.85943422-b)^2-r^2\right)(-16.45099311+2a)+\frac{2}{21}\left((8.252460390-a)^2+(-0.500087624-b)^2-r^2\right)(-16.50492078+2a)+\frac{2}{21}\left((-2.573609023-a)^2+(-2.548154439-b)^2-r^2\right)(5.147218046+2a)+\frac{2}{21}\left((6.217213677-a)^2+(-2.005216909-b)^2-r^2\right)(-12.43442735+2a)+\frac{2}{21}\left((-5.376771502-a)^2+(-0.648554025-b)^2-r^2\right)(10.75354300+2a)+\frac{2}{21}\left((1.957880079-a)^2+(-3.482722737-b)^2-r^2\right)(-3.915760158+2a) \tag{6}$$

$$+\frac{2}{21}\left((9.611841272-a)^2+(1.296996009-b)^2-r^2\right)(-19.22368254+2\,a)+\frac{2}{21}\left((8.583836714-a)^2+(-0.279452485-b)^2-r^2\right)(-17.16767343+2\,a)+\frac{2}{21}\left((0.8999904708-a)^2+(15.74576754-b)^2-r^2\right)(-1.799980942+2\,a)$$

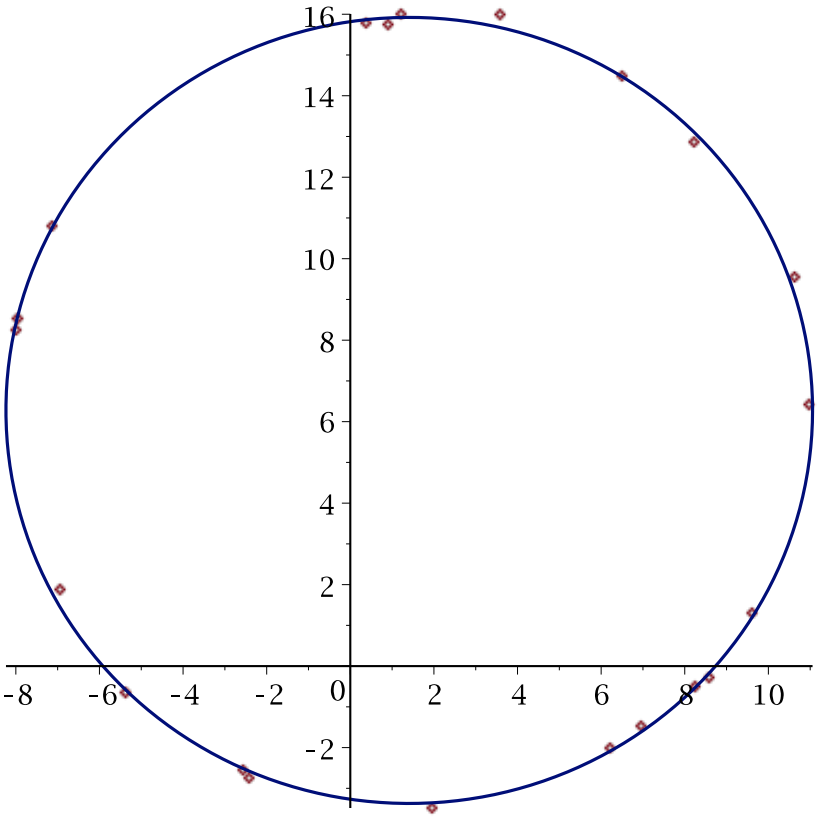
```
> saul := solve( {diff(H(a, b, r, data), a) = 0,
                  diff(H(a, b, r, data), b) = 0,
                  diff(H(a, b, r, data), r) = 0})
```

$$saul := \{a = 1.759554621,\ b = 6.101309227,\ r = 0.\},\ \{a = 4.425622437 + 1.577999976\,I,\ b = 6.052756424 - 14.09636695\,I,\ r = 0.\},\ \{a = 2.095285400 + 13.33705866\,I,\ b = 7.935958431 + 0.7960221885\,I,\ r = 0.\},\ \{a = 2.095285400 - 13.33705866\,I,\ b = 7.935958431 - 0.7960221885\,I,\ r = 0.\},\ \{a = 4.425622437 - 1.577999976\,I,\ b = 6.052756424 + 14.09636695\,I,\ r = 0.\},\ \{a = 1.409800682,\ b = 6.275566340,\ r = 9.646968874\},\ \{a = 1.409800682,\ b = 6.275566340,\ r = -9.646968874\} \quad \textbf{(7)}$$

```
> circ := subs({a = 1.409800682, b = 6.275566340, r = 9.646968874},
                [r·cos(t) + a, r·sin(t) + b, t = 0..2·Pi]);
```

$$circ := [9.646968874\cos(t) + 1.409800682,\ 9.646968874\sin(t) + 6.275566340,\ t = 0..2\,\pi] \quad \textbf{(8)}$$

```
> plot([data, circ], style = [point, line]);
```




```
> saul[6];
```

$$\{a = 1.409800682,\ b = 6.275566340,\ r = 9.646968874\} \quad (9)$$

```
> saul;
```

$$\{a = 1.759554621,\ b = 6.101309227,\ r = 0.\},\ \{a = 4.425622437 + 1.577999976\,I,\ b = 6.052756424 - 14.09636695\,I,\ r = 0.\},\ \{a = 2.095285400 + 13.33705866\,I,\ b = 7.935958431 + 0.7960221885\,I,\ r = 0.\},\ \{a = 2.095285400 - 13.33705866\,I,\ b = 7.935958431 - 0.7960221885\,I,\ r = 0.\},\ \{a = 4.425622437 - 1.577999976\,I,\ b = 6.052756424 + 14.09636695\,I,\ r = 0.\},\ \{a = 1.409800682,\ b = 6.275566340,\ r = 9.646968874\},\ \{a = 1.409800682,\ b = 6.275566340,\ r = -9.646968874\} \quad (10)$$

```
> map(x→x^2, [1, 2, 3, 9]);
```

$$[1, 4, 9, 81] \quad (11)$$

```
> saul[2][3];
```

$$r = 0. \quad (12)$$

```
> saul[7][3];
```

$$r = -9.646968874 \quad (13)$$

```
> subs(saul[7], r);
```

$$-9.646968874 \tag{14}$$

```
> subs(saul[2], r);
```

$$0. \tag{15}$$

```
> goodsol := s→ subs(s, r);
```

$$goodsol := s \rightarrow subs(s, r) \tag{16}$$

```
> map(goodsol, [saul]);
```

$$[0., 0., 0., 0., 0., 9.646968874, -9.646968874] \tag{17}$$

```
> goodsol := s→ if (subs(s, r) > 0) then true; else false; fi;
```

$$goodsol := s \rightarrow \textbf{if } 0 < subs(s, r) \textbf{ then } true \textbf{ else } false \textbf{ end if} \tag{18}$$

```
> map(goodsol, [saul]);
```

$$[false, false, false, false, false, true, false] \tag{19}$$

```
> select(goodsol, [saul]);
```

$$[\{a = 1.409800682, b = 6.275566340, r = 9.646968874\}] \tag{20}$$

```
> remove(goodsol, [saul]);
```

$$[\{a = 1.759554621, b = 6.101309227, r = 0.\}, \{a = 4.425622437 + 1.577999976\,I, b = 6.052756424 - 14.09636695\,I, r = 0.\}, \{a = 2.095285400 + 13.33705866\,I, b = 7.935958431 + 0.7960221885\,I, r = 0.\}, \{a = 2.095285400 - 13.33705866\,I, b = 7.935958431 - 0.7960221885\,I, r = 0.\}, \{a = 4.425622437 - 1.577999976\,I, b = 6.052756424 + 14.09636695\,I, r = 0.\}, \{a = 1.409800682, b = 6.275566340, r = -9.646968874\}] \tag{21}$$

```
>
>
>
```

automate this.

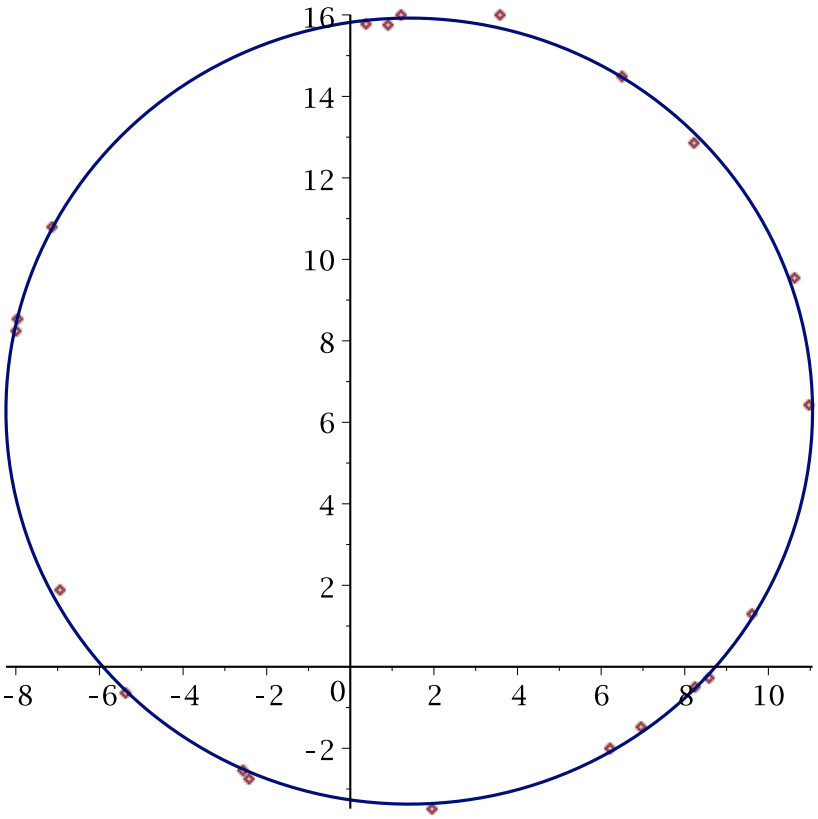
Want FitCirc that finds the circle [r*cos(t)+a,r*sin(t)+b,t=0..2*Pi] given datapoints.

```
> FitCirc := proc(data)
    local v, H, sol, goodsol, good, circ;
    v := (a, b, r, p)→ (p[1] - a)^2 + (p[2] - b)^2 - r^2;
    H := (a, b, r, pts)→ sum(v(a, b, r, pts[i])^2, i = 1..nops(pts))/nops(pts);
    sol := solve( {diff(H(a, b, r, data), a) = 0,
                   diff(H(a, b, r, data), b) = 0,
                   diff(H(a, b, r, data), r) = 0});
    goodsol := s→ if (subs(s, r) > 0) then true; else false; fi;
    good := op(select(goodsol, [sol]));
    circ := subs(good, [r·cos(t) + a, r·sin(t) + b, t = 0..2·Pi]);
  end:
```

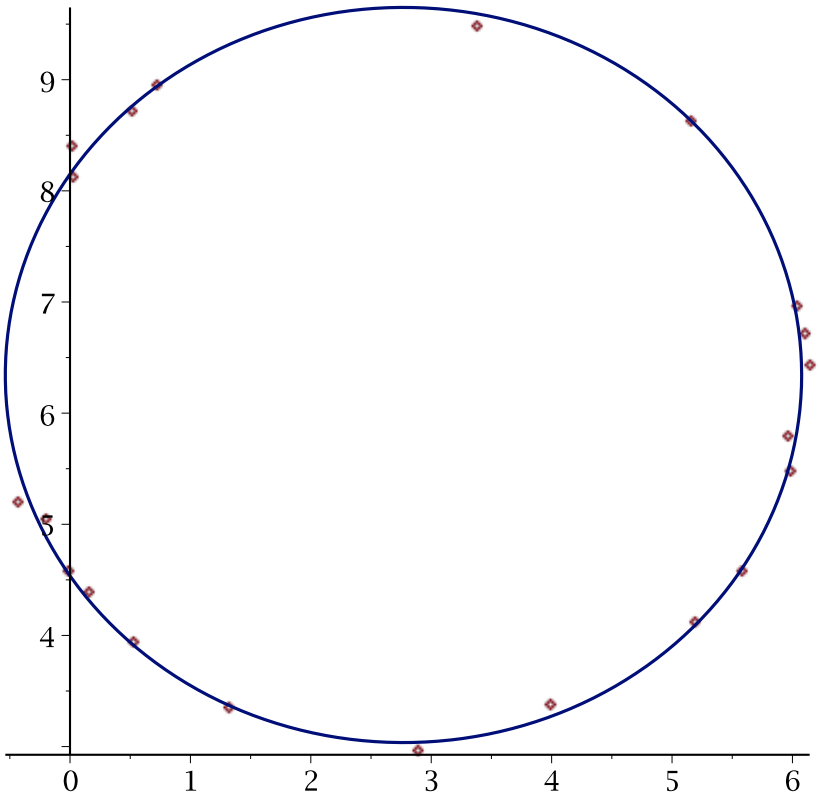
```
> FitCirc(data);
```

$$[9.646968874\cos(t) + 1.409800682,\ 9.646968874\sin(t) + 6.275566340,\ t = 0..2\pi] \quad \textbf{(22)}$$

```
> plot([data, FitCirc(data)], style = [point, line]);
```




```
> data := circle_pts( ) : plot([data, FitCirc(data)], style = [point, line]);
```


9
8
7
6
5
4
0
1
2
3
4
5
6