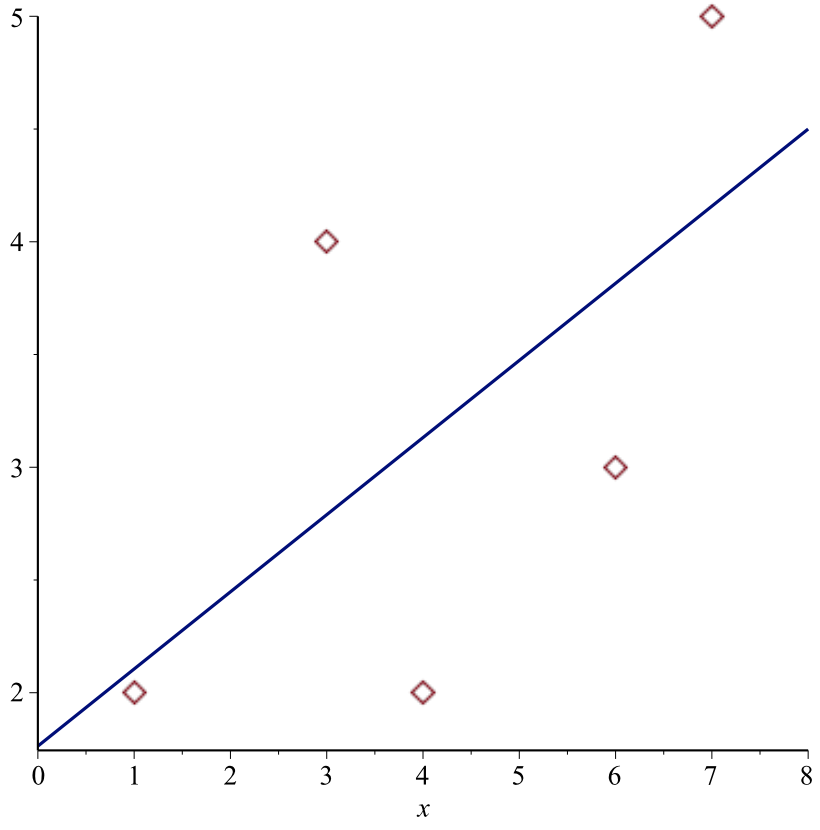
```
> data := [[1, 2], [3, 4], [4, 2], [6, 3], [7, 5]];
```

$$data := [[1, 2], [3, 4], [4, 2], [6, 3], [7, 5]] \quad (1)$$

```
> with(CurveFitting) :
> mline := LeastSquares(data, x);
```

$$mline := \frac{67}{38} + \frac{13}{38}\, x \quad (2)$$

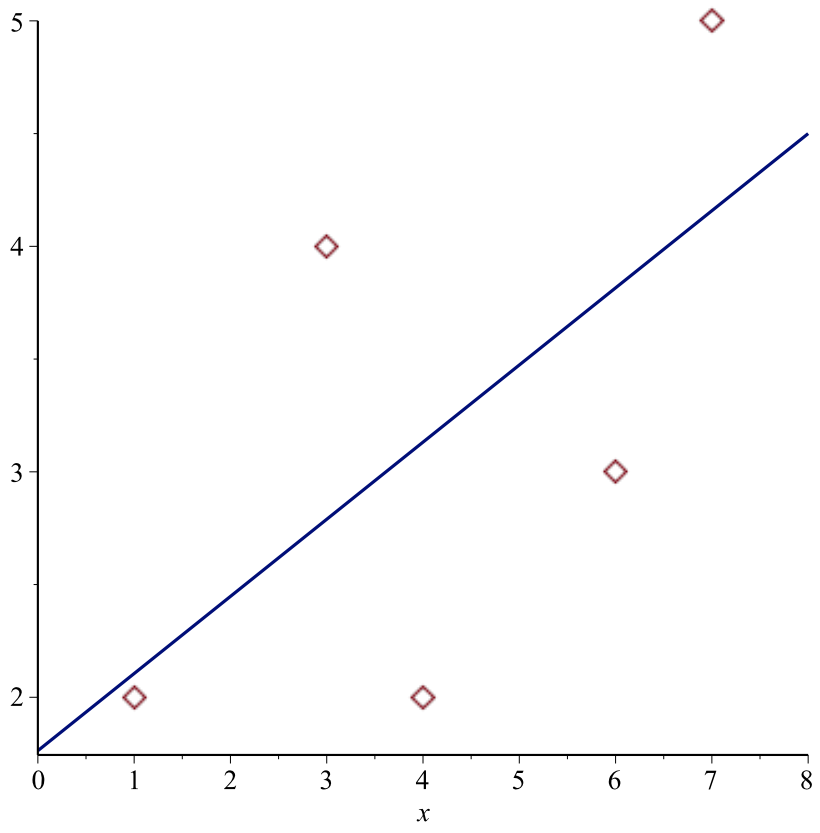
```
> plot([data, mline], x = 0..8, style = [point, line], symbolsize = 20);
```




```
> OPTS := x = 0..8, style = [point, line$3], symbolsize = 20;
```

$$OPTS := x = 0..8, style = [point, line, line, line], symbolsize = 20 \quad (3)$$

```
> plot([data, mline], OPTS)
```




```
> dp := (pt,f)->(pt[2]-f(pt[1]))^2;
```

$$dp := (pt,f) \to \left(pt_2 - f\left(pt_1\right)\right)^2 \tag{4}$$

```
> dp([1,3],x->x^2);
```

$$4 \tag{5}$$

```
> dist := (data,f)-> sum(dp(data[i],f), i=1..nops(data))/nops(data);
```

$$dist := (data,f) \to \frac{\sum_{i=1}^{nops(data)} dp\left(data_i,f\right)}{nops(data)} \tag{6}$$

```
> dist(data,x->x^2);
```

$$\frac{3247}{5} \tag{7}$$

```
> dist(data,x->x/2+1);
```

$$\frac{19}{20} \tag{8}$$

```
> dist(data, x -> x·5/8 + 1);
```

$$\frac{439}{320} \tag{9}$$

```
> ld := dist(data, x -> m·x + b);
```

$$ld := \frac{1}{5}(-b-m+2)^2 + \frac{1}{5}(-b-3m+4)^2 + \frac{1}{5}(-b-4m+2)^2 + \frac{1}{5}(-b-6m+3)^2 + \frac{1}{5}(-b-7m+5)^2 \tag{10}$$

```
> diff(ld, m);
```

$$\frac{42}{5}b + \frac{222}{5}m - 30 \tag{11}$$

```
> diff(ld, b);
```

$$2b + \frac{42}{5}m - \frac{32}{5} \tag{12}$$

```
> solve({diff(ld, m) = 0, diff(ld, b) = 0});
```

$$\left\{b = \frac{67}{38}, m = \frac{13}{38}\right\} \tag{13}$$

```
> subs(%, m·x + b);
```

$$\frac{67}{38} + \frac{13}{38}x \tag{14}$$

```
> mline;
```

$$\frac{67}{38} + \frac{13}{38}x \tag{15}$$

```
> dp := (pt,f) -> (pt[2] - f(pt[1]))^2:
> dist := (data,f) -> sum(dp(data[i],f), i = 1..nops(data))/nops(data):
> func := x -> m·x + b;
```

$$func := x \to m x + b \tag{16}$$

```
> subs(solve({diff(dist(data,func), b) = 0,
      diff(dist(data,func), m) = 0}),func(x));
```

$$\frac{67}{38} + \frac{13}{38}x \tag{17}$$

```
> f:=x->x^2;
```

$$f := x \to x^2 \tag{18}$$

```
> f(3);
```

$$9 \tag{19}$$

```
> g :=proc(x)
  x^2;
 end;
```

$$g := \mathbf{proc}(x)\ x\text{^}2\ \mathbf{end\ proc} \tag{20}$$

```
> g(3);
```

$$9 \tag{21}$$

```
> LSQ :=proc(data, func)
```

```
    dp := (pt,f) -> (pt[2] - f(pt[1]))^2:
    dist := (data,f) -> sum(dp(data[i],f), i=1..nops(data))/nops(data):
    subs(solve({diff(dist(data,func),b)=0,
      diff(dist(data,func),m)=0}),func(x));
  end:
```

Warning, `dp` is implicitly declared local to procedure `LSQ`

Warning, `dist` is implicitly declared local to procedure `LSQ`

```
> LSQ(data, x->m*x+b);
```

$$\frac{67}{38} + \frac{13}{38}\,x \tag{22}$$

```
> mparab := LSQ(data, x->m*x^2+b);
```

$$mparab := \frac{113}{2618}\,x^2 + \frac{5869}{2618} \tag{23}$$

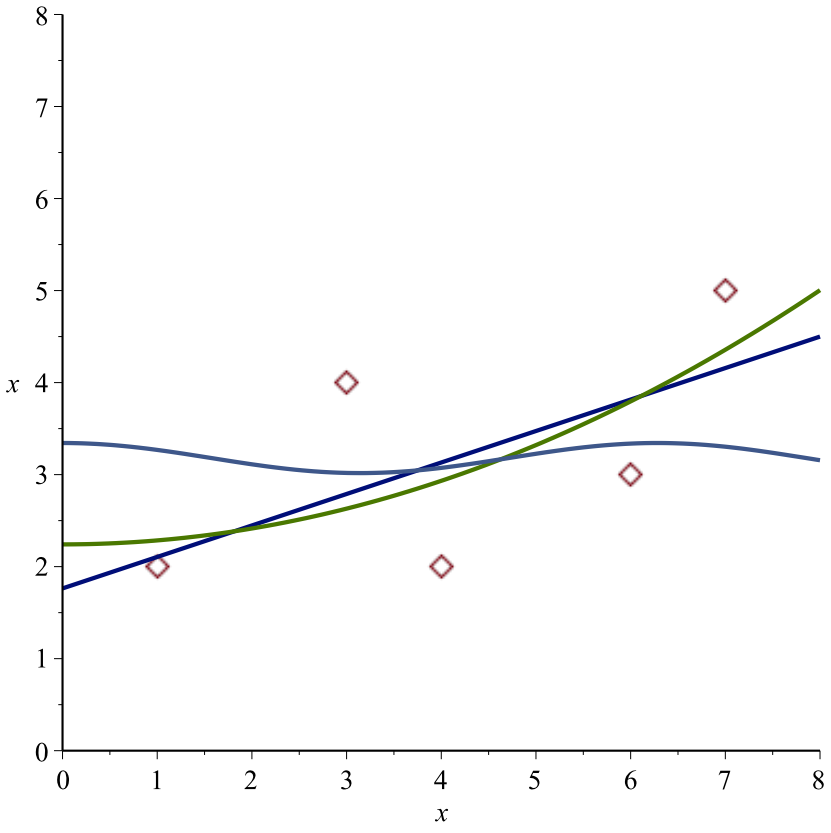
```
> mcos := LSQ(data, x->m*cos(x)+b);
```

$$mcos := -\frac{1}{2}\,((6\cos(1) - 4\cos(3) + 6\cos(4) + \cos(6) - 9\cos(7))\cos(x))\Big/(2\cos(1)^2 \tag{24}$$
$$- \cos(1)\cos(3) - \cos(1)\cos(4) - \cos(1)\cos(6) - \cos(1)\cos(7) + 2\cos(3)^2$$
$$- \cos(3)\cos(4) - \cos(3)\cos(6) - \cos(3)\cos(7) + 2\cos(4)^2 - \cos(4)\cos(6)$$
$$- \cos(4)\cos(7) + 2\cos(6)^2 - \cos(6)\cos(7) + 2\cos(7)^2) + \frac{1}{2}\,(14\cos(1)^2$$
$$- 6\cos(1)\cos(3) - 4\cos(1)\cos(4) - 5\cos(1)\cos(6) - 7\cos(1)\cos(7) + 12\cos(3)^2$$
$$- 6\cos(3)\cos(4) - 7\cos(3)\cos(6) - 9\cos(3)\cos(7) + 14\cos(4)^2 - 5\cos(4)\cos(6)$$
$$- 7\cos(4)\cos(7) + 13\cos(6)^2 - 8\cos(6)\cos(7) + 11\cos(7)^2)\Big/(2\cos(1)^2$$
$$- \cos(1)\cos(3) - \cos(1)\cos(4) - \cos(1)\cos(6) - \cos(1)\cos(7) + 2\cos(3)^2$$
$$- \cos(3)\cos(4) - \cos(3)\cos(6) - \cos(3)\cos(7) + 2\cos(4)^2 - \cos(4)\cos(6)$$
$$- \cos(4)\cos(7) + 2\cos(6)^2 - \cos(6)\cos(7) + 2\cos(7)^2)$$

```
> evalf(mcos);
```

$$0.1634069400\cos(x) + 3.180040210 \tag{25}$$

```
> plot([data, mline, mparab, mcos], x=0..8, OPTS, thickness=2);
```



> $LSQ(data, x \to m \cdot \cos(x + b))$;

$$0 \quad (26)$$

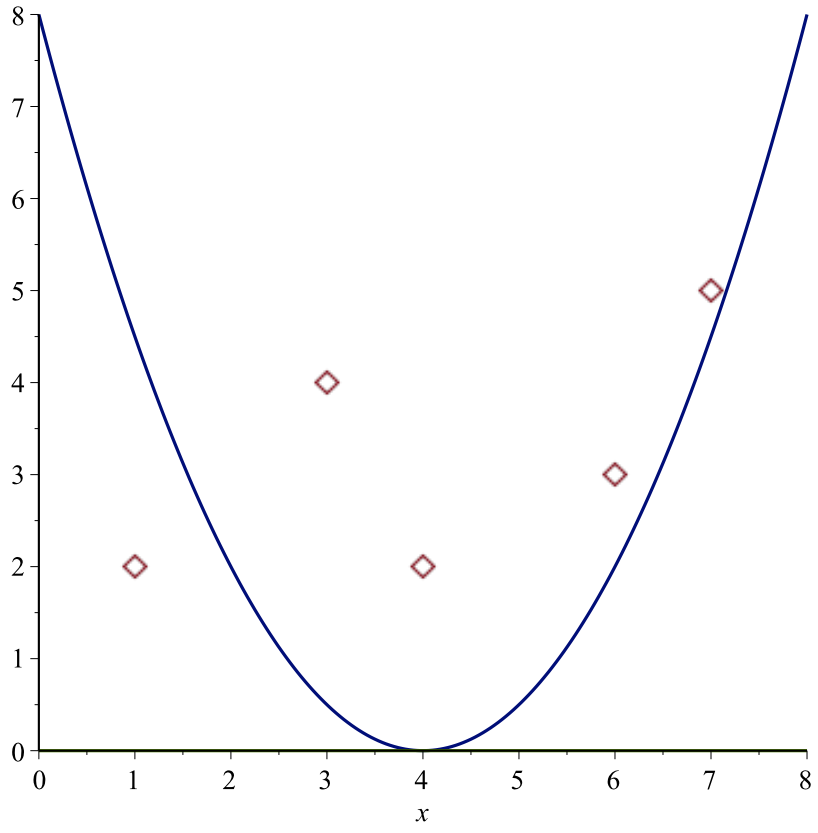
> $LSQ(data, x \to m \cdot (x + b)^2)$;

$$0 \quad (27)$$

> $LSQ(data, x \to m + (x + b)^2)$;

$$-\frac{8441}{5776} + \left(x - \frac{295}{76}\right)^2 \quad (28)$$

> $plot\left(\left[data, \frac{1}{2} \cdot (x-4)^2, 0\right], OPTS\right)$;



> $dist\left(data, x \to \frac{1}{2} \cdot (x-4)^2\right); evalf(\%)$

$$\frac{19}{4}$$

$$4.750000000 \quad \textbf{(29)}$$

> $dist(data, 0); evalf(\%);$

$$\frac{58}{5}$$

$$11.60000000 \quad \textbf{(30)}$$

> $LeastSquares(data, x, curve = m \cdot x^2 + b);$

$$\frac{113}{2618} x^2 + \frac{5869}{2618} \quad \textbf{(31)}$$

> $LeastSquares(data, x, curve = m \cdot (x-b)^2);$

Error, (in CurveFitting:-LeastSquares) curve to fit is not linear in the parameters

> $mbdist := dist(data[1..2], x \to m \cdot (x-b)^2);$

$$mbdist := \frac{1}{2}\left(2 - m\,(1-b)^2\right)^2 + \frac{1}{2}\left(4 - m\,(3-b)^2\right)^2 \quad \textbf{(32)}$$

```
> bdiff := diff(mbdist, b);
```

$$bdiff := 2\left(2 - m\,(1-b)^2\right) m\,(1-b) + 2\left(4 - m\,(3-b)^2\right) m\,(3-b) \tag{33}$$

```
> mdiff := diff(mbdist, m);
```

$$mdiff := -\left(2 - m\,(1-b)^2\right)(1-b)^2 - \left(4 - m\,(3-b)^2\right)(3-b)^2 \tag{34}$$

```
> solve({bdiff = 0, mdiff = 0});
```

$$\left\{b = RootOf\left(3\_Z^2 - 14\_Z + 19\right), m = 0\right\}, \{b = 1, m = 1\}, \left\{b = 3, m = \frac{1}{2}\right\}, \left\{b = RootOf\left(\_Z^2 + 2\_Z - 7\right), m = \frac{1}{2} RootOf\left(\_Z^2 + 2\_Z - 7\right) + 2\right\} \tag{35}$$

```
> evalf(%);
```

$$\{b = 2.333333333 + 0.9428090416\,I, m = 0.\}, \{b = 1., m = 1.\}, \{b = 3., m = 0.5000000000\}, \{b = 1.828427125, m = 2.914213562\} \tag{36}$$

```
> mbdist := dist(data, x -> m·(x - b)^2) :
  solve({diff(mbdist, m) = 0, diff(mbdist, b) = 0});
```

$$\left\{b = \frac{3}{2} RootOf\left(4\_Z^2 - 25\_Z + 47\right), m = 0\right\}, \left\{b = RootOf\left(13\_Z^4 + 21\_Z^3 - 1533\_Z^2 + 7117\_Z - 9084\right), m = -\frac{471003}{119546894} RootOf\left(13\_Z^4 + 21\_Z^3 - 1533\_Z^2 + 7117\_Z - 9084\right)^3 - \frac{1324869}{59773447} RootOf\left(13\_Z^4 + 21\_Z^3 - 1533\_Z^2 + 7117\_Z - 9084\right)^2 + \frac{23069119}{59773447} RootOf\left(13\_Z^4 + 21\_Z^3 - 1533\_Z^2 + 7117\_Z - 9084\right) - \frac{30471057}{59773447}\right\} \tag{37}$$

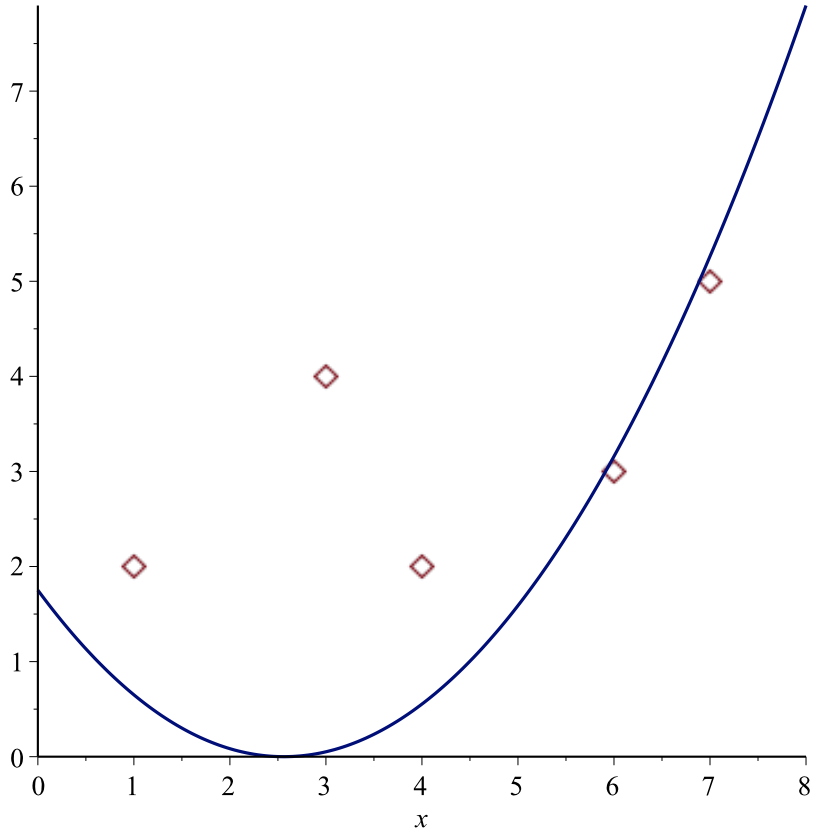
```
> evalf(%);
```

$$\{b = 4.687500000 + 2.113017688\,I, m = 0.\}, \{b = 2.56087139152897, m = 0.267047019687719\} \tag{38}$$

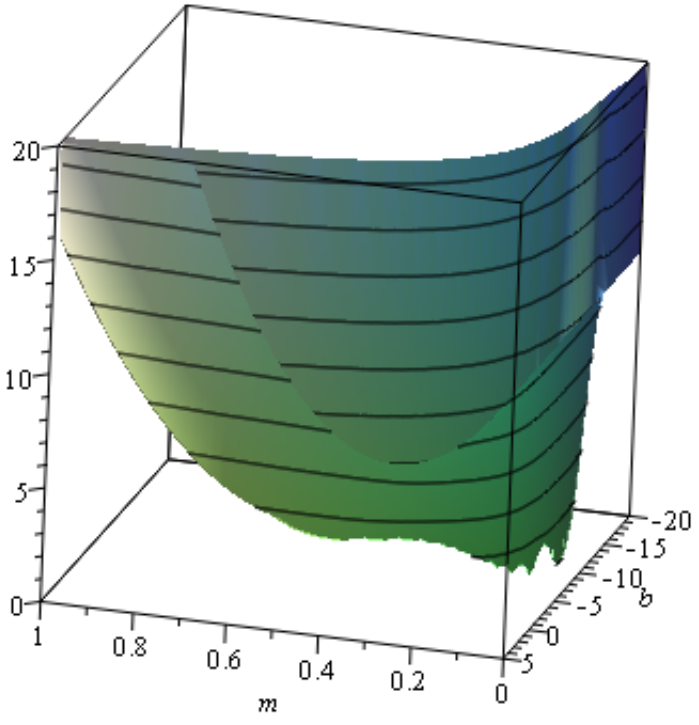
```
> mq := subs(%[2], m·(x - b)^2);
```

$$mq := 0.267047019687719\,(x - 2.56087139152897)^2 \tag{39}$$

```
> plot([data, mq], OPTS);
```



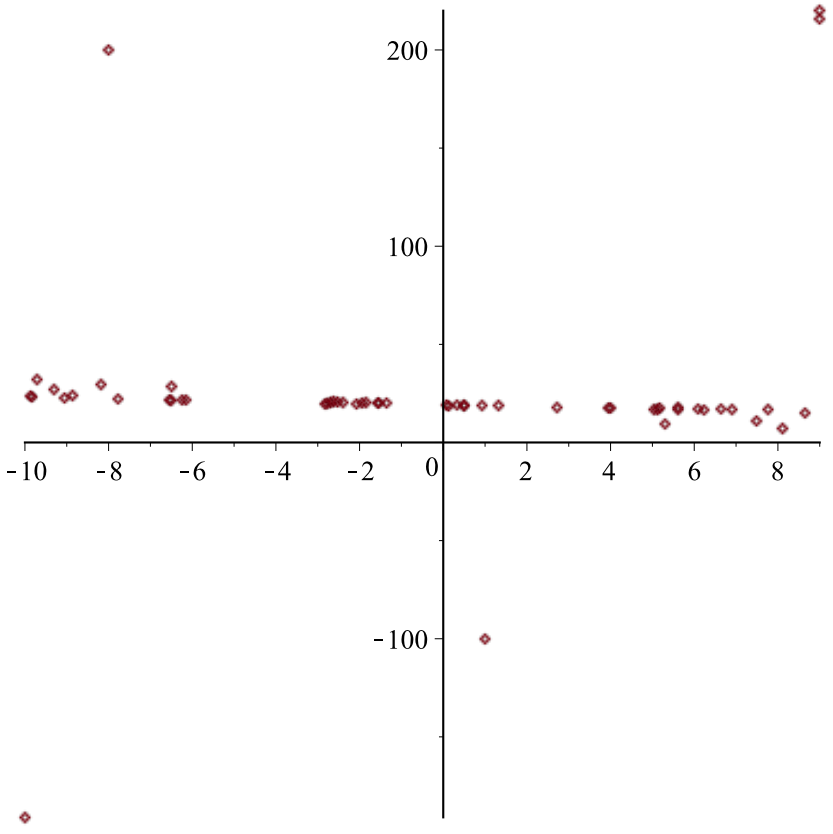

```
> with(plots):
> plot3d(dist(data, x->m*(x-b)^2), m=0..1, b=-20..5, style=patchcontour, view=0..20);
```



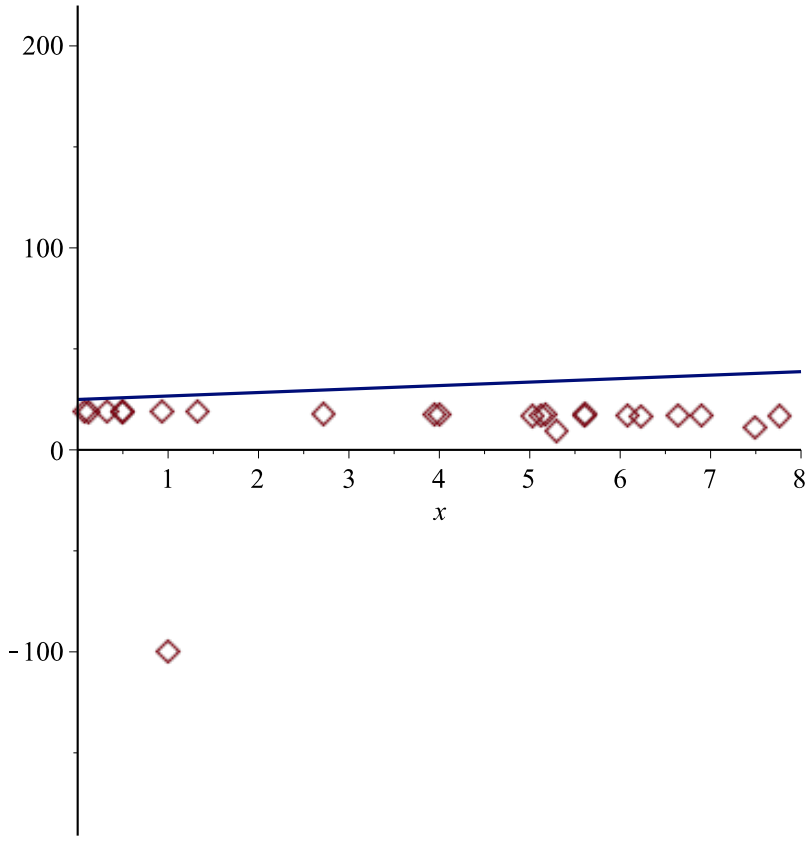

```
> mbdist := dist(data, x→m·(x−b)^2) :
  fsolve({diff(mbdist, m)=0, diff(mbdist, b)=0}, {m=0.1, b=-10});
                    {b = 2.560871392, m = 0.2670470197}                    (40)

> ?fsolve

> bdata := [[8.65124763, 15.05801139], [.12218972, 18.71889232],
  [.7461504e-1, 19.08078948], [-1.559782888, 19.98371172],
  [-9.827561842, 23.06848486], [.32311207, 19.14564218],
  [-2.391913592, 20.30217888], [3.94617018, 17.43607480],
  [-2.830376788, 19.63133334], [-1.358328420, 20.05752868],
  [6.23095437, 16.43323636], [5.61148003, 17.81101690],
  [-2.625640890, 20.88674163], [-2.690509364, 20.36160632],
  [-8.857403420, 23.84930474], [-2.780958614, 19.81177275],
  [.49220217, 18.73849995], [-6.543130054, 21.51666222],
  [6.08427501, 16.97527217], [-6.242938578, 21.72698882],
  [-6.514007062, 21.45289388], [7.76133332, 16.75089278],
  [-9.054911088, 22.54190263], [5.03304675, 16.82722949],
  [-9.869721387, 23.70277711], [-7.770840930, 22.16242030],
  [-1.948518384, 20.06695018], [4.00205027, 17.47005315],
  [5.17321040, 17.55005418],
```

```
[-2.076084772, 19.72089223], [-1.550062590, 20.21390427],
[-6.143925098, 21.65680593], [5.60762551, 16.84921688],
[6.64081052, 17.04952989],
[2.71911796, 17.75913317], [.49583125, 18.89613761],
[5.12446722, 16.91928245], [-1.852442276, 20.25908204],
[6.90280682, 16.84990814], [-2.532240154, 20.72224765],
[1.32201814, 18.91342503], [.93254506, 18.90640481],
[-8.184966738, 29.44895089], [-9.307860036, 27.09063691],
[-9.710001259, 32.12397498], [-6.499868778, 28.48508601],
[7.492109388, 11.06831268], [5.295254201, 9.403916851],
[8.105629482, 7.166928438], [9, 215.6599449],
[-10, -191], [-8, 200], [9, 220], [1, -100]]:
```

```
> plot(bdata, style=point);
```




```
> plot([bdata, LeastSquares(bdata, x)], OPTS);
```




```
> bdata := ick;
```

$$bdata := ick \tag{41}$$

```
> plot( [bdata, LeastSquares(bdata, x) ], OPTS);
```

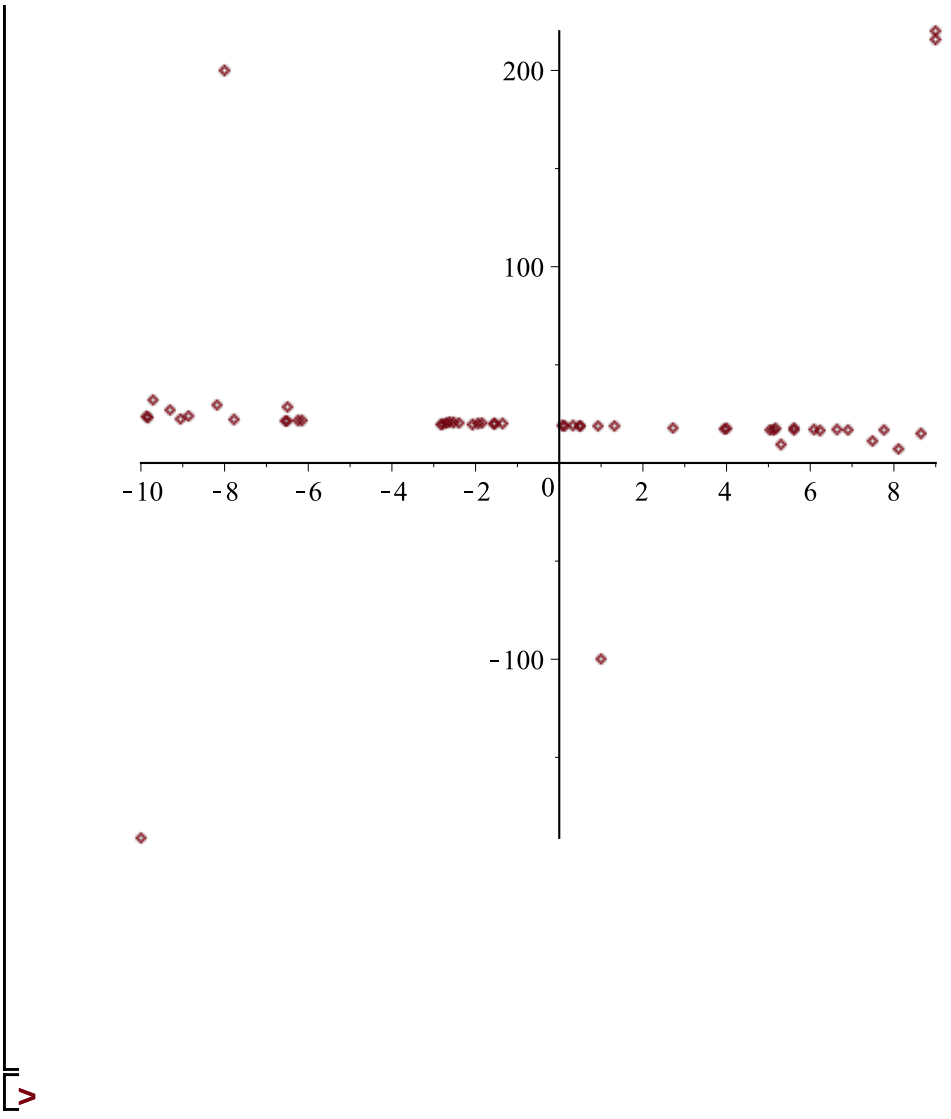
Error, (in CurveFitting:-LeastSquares) data points not in recognizable format

```
> read("/home/scott/class/current/www/problems/bdata.txt");
```

$$bdata := [[8.65124763, 15.05801139], [0.12218972, 18.71889232], [0.07461504, 19.08078948], [-1.559782888, 19.98371172], [-9.827561842, 23.06848486], [0.32311207, 19.14564218], [-2.391913592, 20.30217888], [3.94617018, 17.43607480], [-2.830376788, 19.63133334], [-1.358328420, 20.05752868], [6.23095437, 16.43323636], [5.61148003, 17.81101690], [-2.625640890, 20.88674163], [-2.690509364, 20.36160632], [-8.857403420, 23.84930474], [-2.780958614, 19.81177275], [0.49220217, 18.73849995], [-6.543130054, 21.51666222], [6.08427501, 16.97527217], [-6.242938578, 21.72698882], [-6.514007062, 21.45289388], [7.76133332, 16.75089278], [-9.054911088, 22.54190263], [5.03304675, 16.82722949], [-9.869721387, 23.70277711], [-7.770840930, 22.16242030], [-1.948518384, 20.06695018], [4.00205027, 17.47005315], [5.17321040, 17.55005418], [-2.076084772, 19.72089223], [-1.550062590, 20.21390427], [-6.143925098, 21.65680593], \tag{42}$$

```
[5.60762551, 16.84921688], [6.64081052, 17.04952989], [2.71911796, 17.75913317],
[0.49583125, 18.89613761], [5.12446722, 16.91928245], [-1.852442276, 20.25908204],
[6.90280682, 16.84990814], [-2.532240154, 20.72224765], [1.32201814, 18.91342503],
[0.93254506, 18.90640481], [-8.184966738, 29.44895089], [-9.307860036,
27.09063691], [-9.710001259, 32.12397498], [-6.499868778, 28.48508601],
[7.492109388, 11.06831268], [5.295254201, 9.403916851], [8.105629482,
7.166928438], [9, 215.6599449], [-10, -191], [-8, 200], [9, 220], [1, -100]]
>
> ReadFromWeb := proc(URL :: string, {printfile :: truefalse := false})
  local n, m, status, webfile, headers;
  status, webfile, headers := HTTP[Get](URL) :
  if ( HTTP[Code](status) ≠ "OK") then
     error(HTTP[Code](status), URL);
  fi;
  # now read the web page
  n := 0 :
  while (n < length(webfile)) do
    m := n;
    ;
    if (printfile) then printf("%s", webfile[m + 1 ..n]); fi;
  od:
 end:
> HTTP[Get]("www.math.sunysb.edu/~scott/mat331.spr14/problems/bdata.txt");
          200, "bdata := [[8.65124763, 15.05801139], [.12218972, 18.71889232],                (43)
                [.7461504e-1, 19.08078948], [-1.559782888, 19.98371172],
                [-9.827561842, 23.06848486], [.32311207, 19.14564218],
                [-2.391913592, 20.30217888], [3.94617018, 17.43607480],
               [-2.830376788, 19.63133334], [-1.358328420, 20.05752868],
                [6.23095437, 16.43323636], [5.61148003, 17.81101690],
               [-2.625640890, 20.88674163], [-2.690509364, 20.36160632],
               [-8.857403420, 23.84930474], [-2.780958614, 19.81177275],
                [.49220217, 18.73849995], [-6.543130054, 21.51666222],
                [6.08427501, 16.97527217], [-6.242938578, 21.72698882],
                [-6.514007062, 21.45289388], [7.76133332, 16.75089278],
                [-9.054911088, 22.54190263], [5.03304675, 16.82722949],
               [-9.869721387, 23.70277711], [-7.770840930, 22.16242030],
                [-1.948518384,20.06695018], [4.00205027, 17.47005315],
                                [5.17321040, 17.55005418],
               [-2.076084772, 19.72089223], [-1.550062590, 20.21390427],
                [-6.143925098, 21.65680593], [5.60762551, 16.84921688],
                                [6.64081052, 17.04952989],
                [2.71911796, 17.75913317], [.49583125, 18.89613761],
                [5.12446722, 16.91928245], [-1.852442276, 20.25908204],
                [6.90280682, 16.84990814], [-2.532240154, 20.72224765],
```

```
                [1.32201814, 18.91342503], [.93254506, 18.90640481],
              [-8.184966738, 29.44895089], [-9.307860036, 27.09063691],
              [-9.710001259, 32.12397498], [-6.499868778, 28.48508601],
              [7.492109388, 11.06831268], [5.295254201, 9.403916851],
                [8.105629482, 7.166928438], [9, 215.6599449],
                  [-10, -191], [-8, 200], [9, 220], [1, -100]];
", table( ["Etag" = ""8f5120-5be-4f2af1bf4e93f"", "Vary" = "Accept-Encoding", "Content-Type"
     = "text/plain", "Server" = "Apache/2.2.22 (Ubuntu)", "Accept-Ranges" = "bytes",
    "Content-Length" = "1470", "Date" = "Thu, 20 Feb 2014 16:12:30 GMT", "Last-Modified"
     = "Tue, 18 Feb 2014 14:30:09 GMT"])
> ReadFromWeb("www.math.sunysb.edu/~scott/mat331.spr14/problems/bdata.txt");
> bdata;
[[8.65124763, 15.05801139], [0.12218972, 18.71889232], [0.07461504, 19.08078948], [          (44)
    -1.559782888, 19.98371172], [-9.827561842, 23.06848486], [0.32311207,
    19.14564218], [-2.391913592, 20.30217888], [3.94617018, 17.43607480], [
    -2.830376788, 19.63133334], [-1.358328420, 20.05752868], [6.23095437,
    16.43323636], [5.61148003, 17.81101690], [-2.625640890, 20.88674163], [
    -2.690509364, 20.36160632], [-8.857403420, 23.84930474], [-2.780958614,
    19.81177275], [0.49220217, 18.73849995], [-6.543130054, 21.51666222], [6.08427501,
    16.97527217], [-6.242938578, 21.72698882], [-6.514007062, 21.45289388],
    [7.76133332, 16.75089278], [-9.054911088, 22.54190263], [5.03304675, 16.82722949],
    [-9.869721387, 23.70277711], [-7.770840930, 22.16242030], [-1.948518384,
    20.06695018], [4.00205027, 17.47005315], [5.17321040, 17.55005418], [-2.076084772,
    19.72089223], [-1.550062590, 20.21390427], [-6.143925098, 21.65680593],
    [5.60762551, 16.84921688], [6.64081052, 17.04952989], [2.71911796, 17.75913317],
    [0.49583125, 18.89613761], [5.12446722, 16.91928245], [-1.852442276, 20.25908204],
    [6.90280682, 16.84990814], [-2.532240154, 20.72224765], [1.32201814, 18.91342503],
    [0.93254506, 18.90640481], [-8.184966738, 29.44895089], [-9.307860036,
    27.09063691], [-9.710001259, 32.12397498], [-6.499868778, 28.48508601],
    [7.492109388, 11.06831268], [5.295254201, 9.403916851], [8.105629482,
    7.166928438], [9, 215.6599449], [-10, -191], [-8, 200], [9, 220], [1, -100]]
> plot(bdata, style=point);
```


200
100
-100
-10
-8
-6
-4
-2
0
2
4
6
8