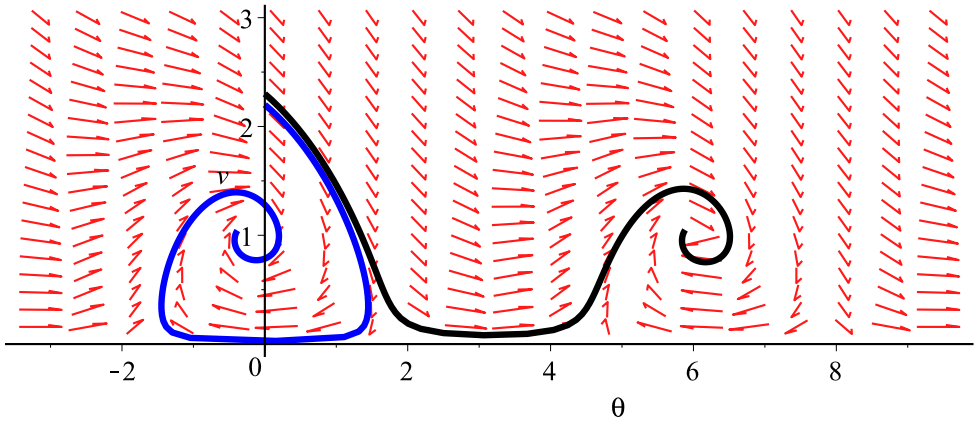
```
>
> with(plots):
  with(DEtools):
> phug:=R->[diff(theta(t),t) = (v(t)^2 - cos(theta(t)))/(v(t)),
            diff(v(t),t)     = -sin(theta(t))-R*v(t)^2];
```

$$phug := R \rightarrow \left[\frac{d}{dt}\,\theta(t) = \frac{v(t)^2 - \cos(\theta(t))}{v(t)},\ \frac{d}{dt}\,v(t) = -\sin(\theta(t)) - R\,v(t)^2 \right] \qquad (1)$$

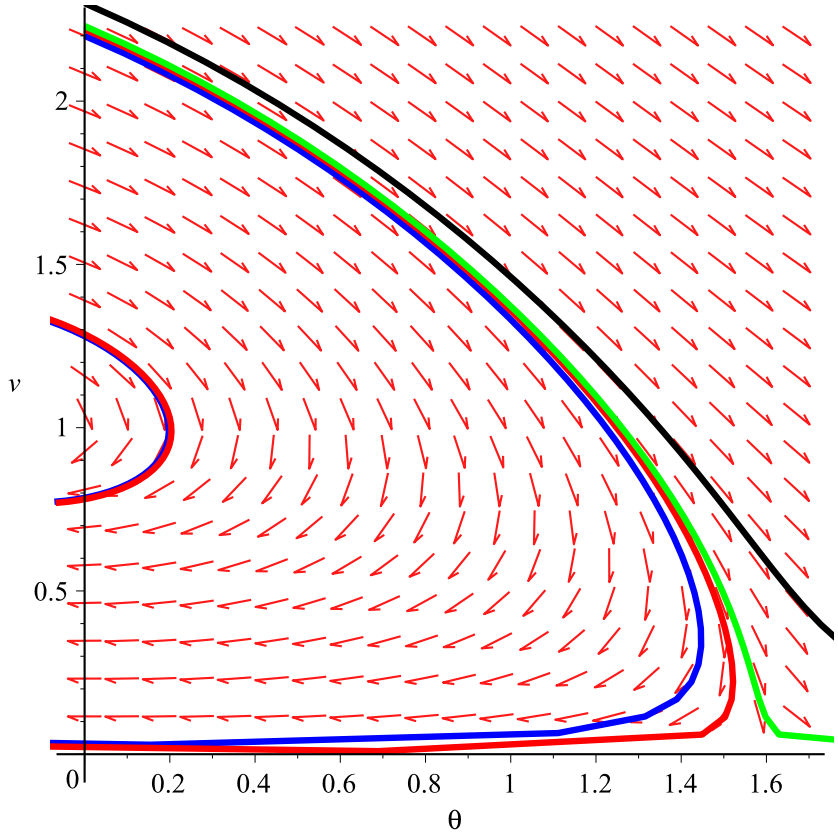
```
> xphug:=R-> [op(phug(R)),
              diff(x(t),t)     = v(t)*cos(theta(t)),
              diff(y(t),t)     = v(t)*sin(theta(t))]:
  xphug(R);
```

$$\left[\frac{d}{dt}\,\theta(t) = \frac{v(t)^2 - \cos(\theta(t))}{v(t)},\ \frac{d}{dt}\,v(t) = -\sin(\theta(t)) - R\,v(t)^2,\ \frac{d}{dt}\,x(t) = v(t)\cos(\theta(t)),\ \frac{d}{dt}\,y(t) = v(t)\sin(\theta(t)) \right] \qquad (2)$$

```
> DEplot(phug(0.2), [theta,v], t=0..8,
       [[v(0)=2.3,theta(0)=0], [v(0)=2.2,theta(0)=0]],
        theta=-Pi..3*Pi, v=0..3,
       linecolor=[black,blue], numpoints=150, obsrange=false);
```



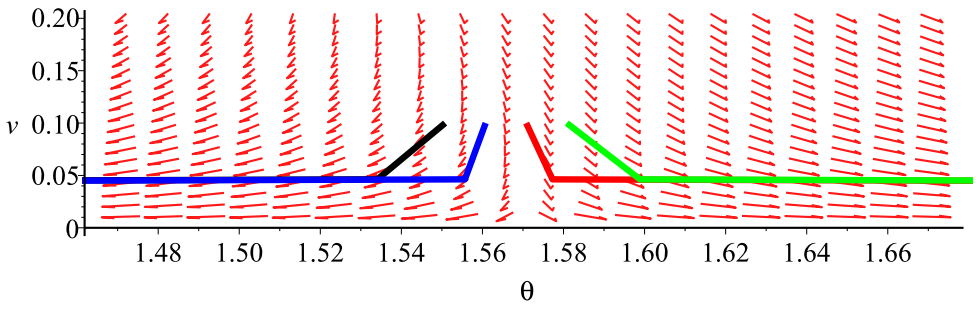

```
> DEplot(phug(0.2), [theta,v], t=0..8,
       [[v(0)=2.3,theta(0)=0], [v(0)=2.2,theta(0)=0],
        [v(0)=2.22,theta(0)=0], [v(0)=2.23,theta(0)=0]],
        theta=0..Pi/2+.1, v=0..2.2,
       linecolor=[black,blue, red, green], numpoints=150,
  obsrange=false);
```



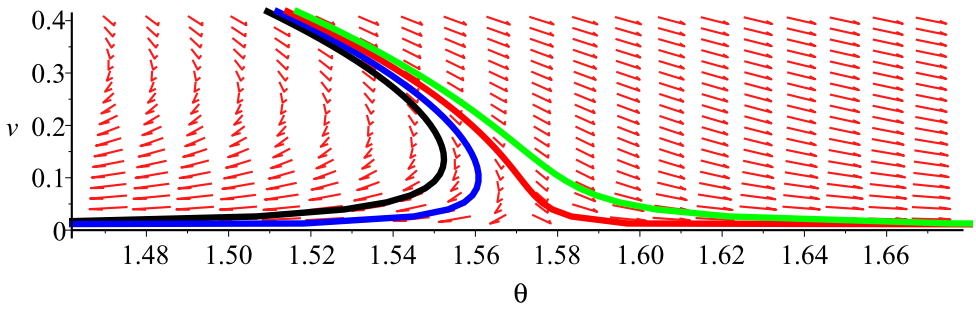
Can see that initial condition which stalls has initial velocity between 2.2 and 2.3, angle 0. Can keep refining if I want.

Can do better by starting near the behavior I want.

```
> DEplot(phug(0.2), [theta,v], t=0..8,
       [[v(0)=0.1,theta(0)=Pi/2-.02],
        [v(0)=0.1,theta(0)=Pi/2-.01],
        [v(0)=0.1,theta(0)=Pi/2],
        [v(0)=0.1,theta(0)=Pi/2+.01]],
       theta=Pi/2-.1..Pi/2+.1, v=0..0.2,
      linecolor=[black,blue, red, green], numpoints=150,
  obsrange=false);
```




```
> DEplot(phug(0.2), [theta,v], t=-1..1,
      [[v(0)=0.1,theta(0)=Pi/2-.02],
       [v(0)=0.1,theta(0)=Pi/2-.01],
       [v(0)=0.1,theta(0)=Pi/2],
       [v(0)=0.1,theta(0)=Pi/2+.01]],
      theta=Pi/2-.1..Pi/2+.1, v=0..0.4,
      linecolor=[black,blue, red, green], numpoints=150,
  obsrange=false);
```




```
> VectorCalculus[Jacobian]([(v^2-cos(theta))/v, -sin(theta)-R*v^2],
  [theta,v]);
```

$$\begin{bmatrix} \frac{\sin(\theta)}{v} & 2-\frac{v^2-\cos(\theta)}{v^2} \\ -\cos(\theta) & -2\,R\,v \end{bmatrix} \tag{3}$$

```
> eval(%,{theta=Pi/2,v=0,R=0.2});
```

Error, numeric exception: division by zero

```
> VectorCalculus[Jacobian]([(v^2-cos(theta)), v*(-sin(theta)-R*v^2)
  ], [theta,v]);
```

$$\begin{bmatrix} \sin(\theta) & 2\,v \\ -v\cos(\theta) & -\sin(\theta)-3\,R\,v^2 \end{bmatrix} \tag{4}$$

```
> A:=eval(%,{theta=Pi/2,v=0,R=0.2});
```

$$A := \begin{bmatrix} 1 & 0 \\ 0 & -1. \end{bmatrix} \tag{5}$$

```
> with(LinearAlgebra):
```

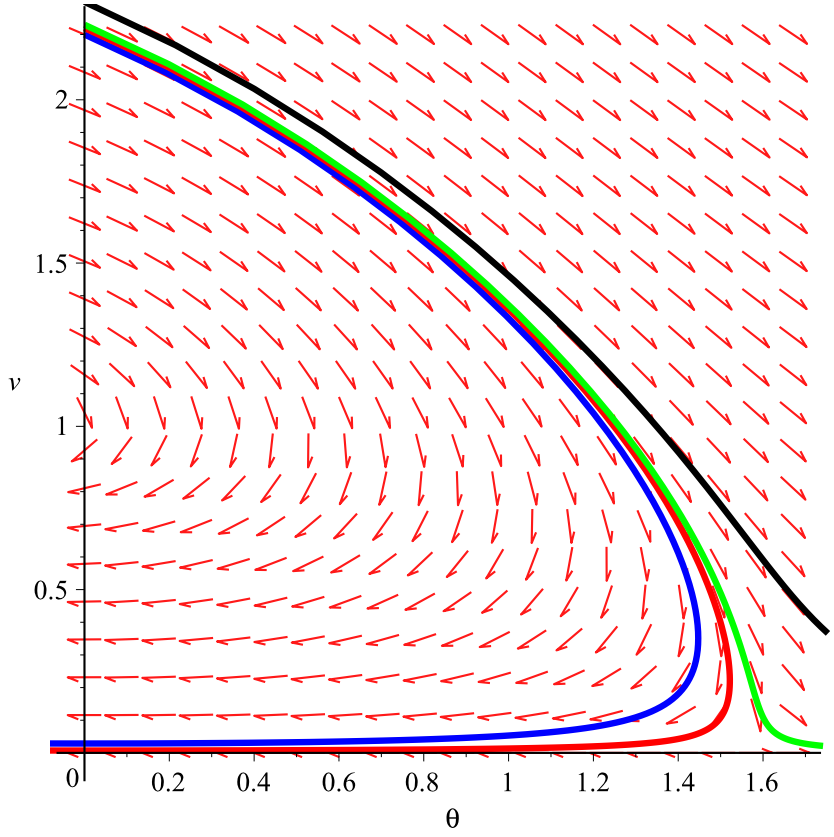
```
> Eigenvectors(A);
```

$$\begin{bmatrix} 1.+0.\,I \\ -1.+0.\,I \end{bmatrix}, \begin{bmatrix} 1.+0.\,I & 0.+0.\,I \\ 0.+0.\,I & 1.+0.\,I \end{bmatrix} \tag{6}$$

```
> vphug:=R->[diff(theta(t),t) = (v(t))^2 - cos(theta(t)),
             diff(v(t),t)     = v(t)*(-sin(theta(t))-R*v(t)^2)];
```

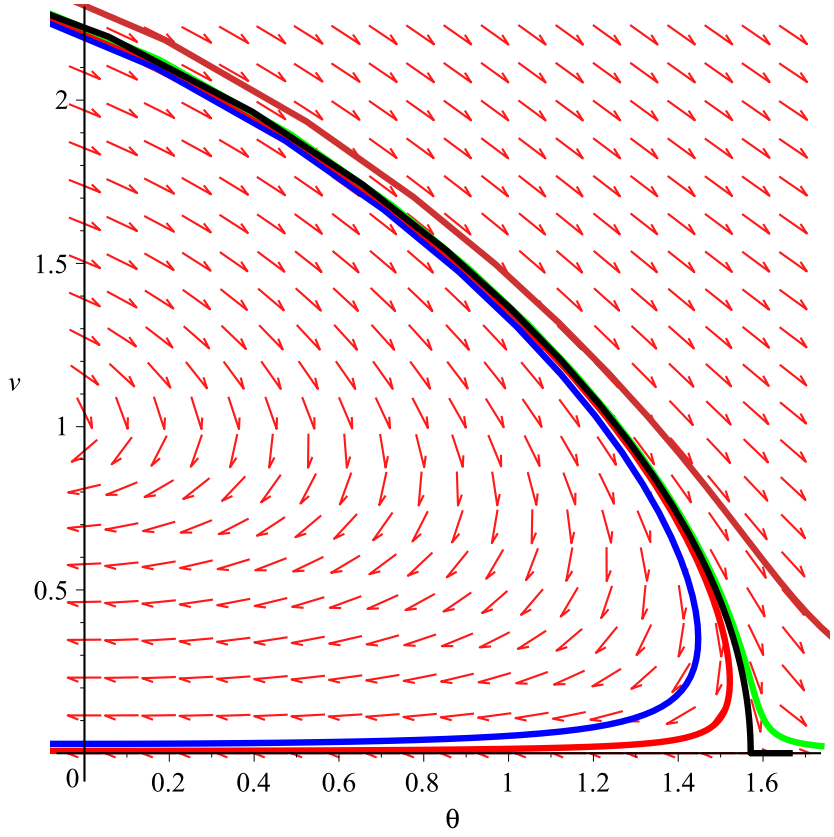
$$vphug := R \rightarrow \left[\frac{d}{dt}\theta(t) = v(t)^2 - \cos(\theta(t)),\ \frac{d}{dt} v(t) = v(t)\left(-\sin(\theta(t)) - R\,v(t)^2\right) \right] \tag{7}$$

```
> DEplot(vphug(0.2), [theta,v], t=0..8,
      [[v(0)=2.3,theta(0)=0], [v(0)=2.2,theta(0)=0],
       [v(0)=2.22,theta(0)=0], [v(0)=2.23,theta(0)=0]],
      theta=0..Pi/2+.1, v=0..2.2,
      linecolor=[black,blue, red, green], numpoints=150,
  obsrange=false);
```




```
> DEplot(vphug(0.2), [theta,v], t=-7..8,
      [[v(0)=2.3,theta(0)=0], [v(0)=2.2,theta(0)=0],
       [v(0)=2.22,theta(0)=0], [v(0)=2.23,theta(0)=0],
       [v(0)=0.01,theta(0)=Pi/2]],  # This is nearly stall.
      theta=0..Pi/2+.1, v=0..2.2,
      linecolor=[orange,blue, red, green, black], numpoints=150,
```

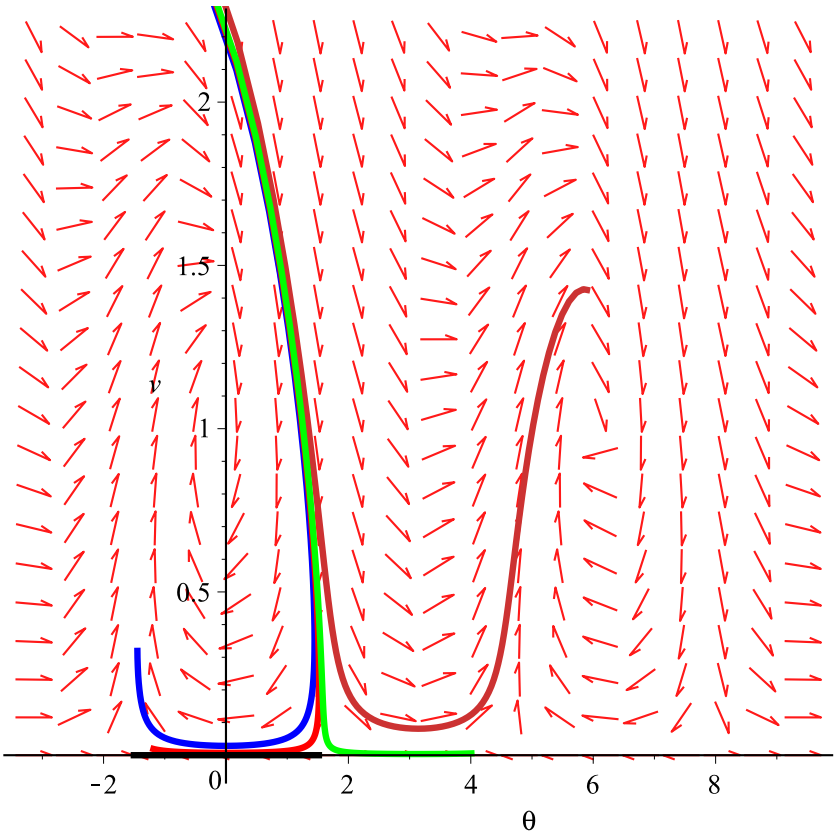
```
  obsrange=false);
```
Warning, plot may be incomplete, the following errors(s) were issued:
  cannot evaluate the solution further left of -.62529895, probably a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
  cannot evaluate the solution further left of -.70626453, probably a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
  cannot evaluate the solution further left of -.68871749, probably a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
  cannot evaluate the solution further left of -.68021571, probably a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
  cannot evaluate the solution further left of -5.8855613, probably a singularity




```
> DEplot(vphug(0.2), [theta,v], t=-7..8,
      [[v(0)=2.3,theta(0)=0], [v(0)=2.2,theta(0)=0],
```

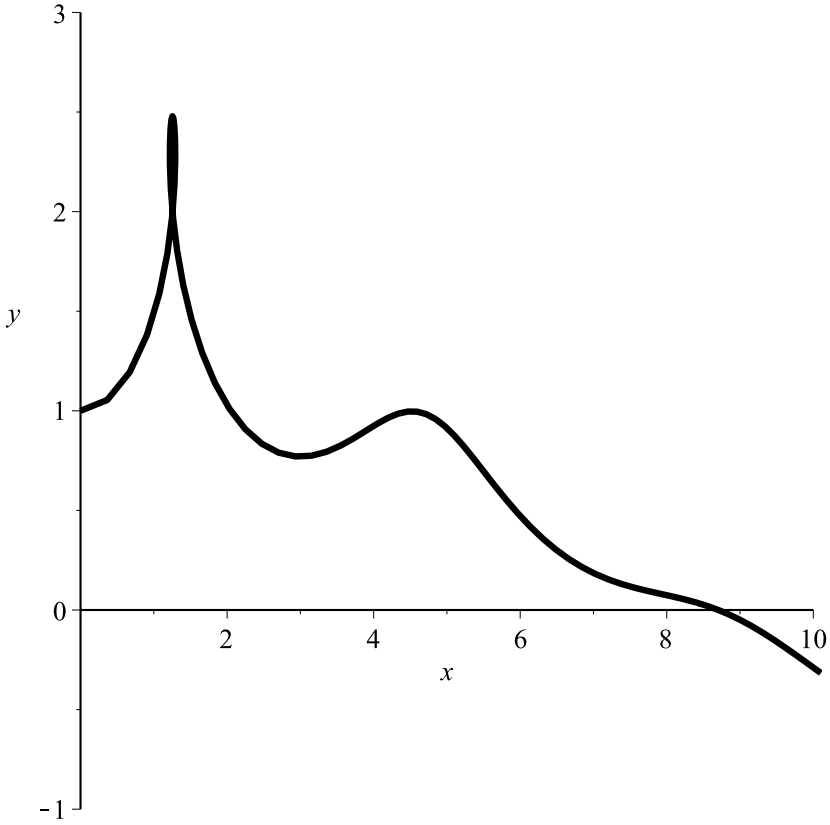
```
        [v(0)=2.22,theta(0)=0], [v(0)=2.23,theta(0)=0],
        [v(0)=0,theta(0)=Pi/2-.1]],  #  This is nearly stall.
       theta=-Pi..3*Pi, v=0..2.2,
      linecolor=[orange,blue, red, green, black], numpoints=150,
  obsrange=false);
```

Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further left of -.62529895, probably a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further left of -.70626453, probably a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further left of -.68871749, probably a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further left of -.68021571, probably a singularity



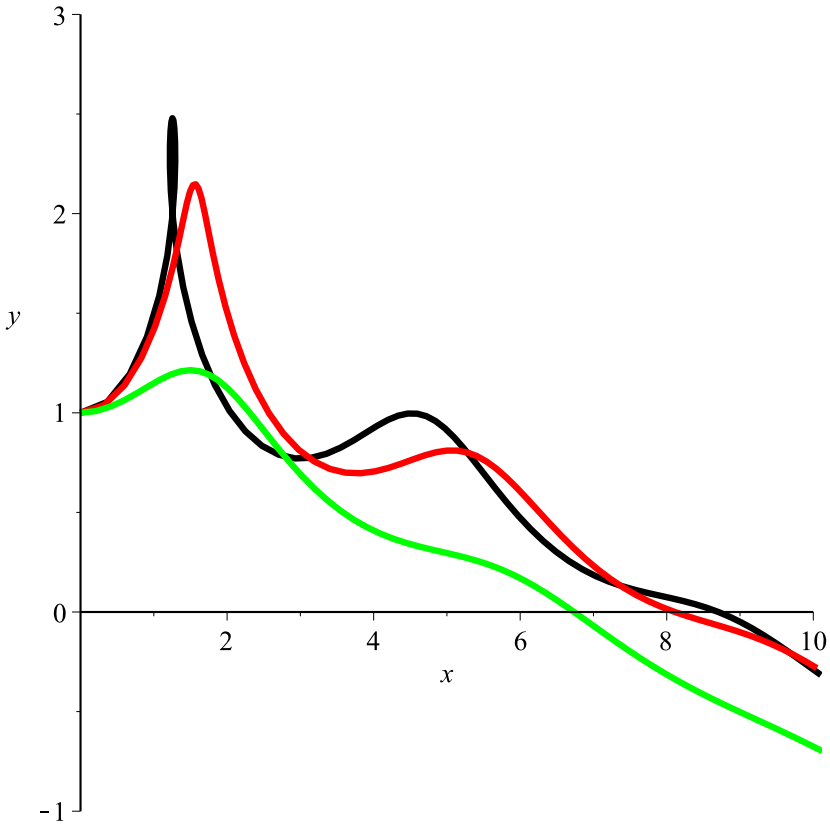

```
> DEplot(xphug(0.2), [theta,v,x,y], t=0..25,
      [[v(0)=2.3,theta(0)=0, x(0)=0, y(0)=1]],
```

```
      theta=-Pi..3*Pi, v=0..2.2,x=0..10, y=-1..3,
      linecolor=black,
    numpoints=150, obsrange=false,
       scene=[x,y]);
```

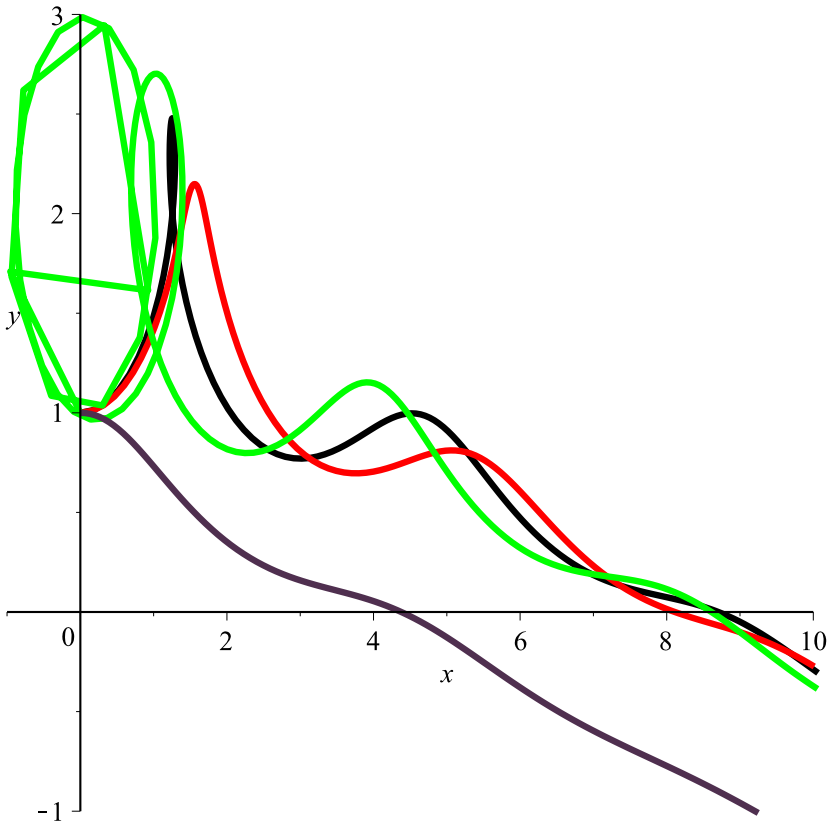


Does increasing velocity make it go further?

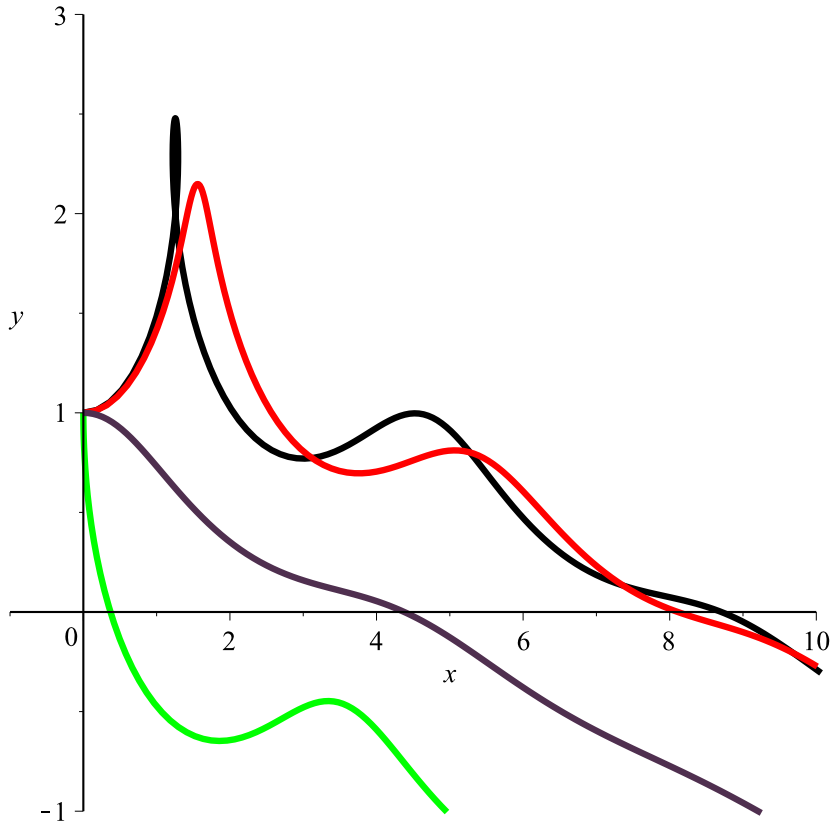
```
> DEplot(xphug(0.2), [theta,v,x,y], t=0..25,
      [[v(0)=2.3,theta(0)=0, x(0)=0, y(0)=1],
       [v(0)=2,theta(0)=0, x(0)=0, y(0)=1],
       [v(0)=1.3,theta(0)=0, x(0)=0, y(0)=1]],
      theta=-Pi..3*Pi, v=0..2.2,x=0..10, y=-1..3,
      linecolor=[black,red, green],
    numpoints=150, obsrange=false,
       scene=[x,y]);
```




```
> DEplot(xphug(0.2), [theta,v,x,y], t=0..25,
      [[v(0)=2.3,theta(0)=0, x(0)=0, y(0)=1],
       [v(0)=2,theta(0)=0, x(0)=0, y(0)=1],
       [v(0)=103,theta(0)=0, x(0)=0, y(0)=1],
       [v(0)=0.8,theta(0)=0, x(0)=0, y(0)=1]],
      theta=-Pi..3*Pi, v=0..2.2,x=-1..10, y=-1..3,
      linecolor=[black,red, green,violet],
     numpoints=300, obsrange=false,
        scene=[x,y]);
```




```
> DEplot(xphug(0.2), [theta,v,x,y], t=0..25,
      [[v(0)=2.3,theta(0)=0, x(0)=0, y(0)=1],
       [v(0)=2,theta(0)=0, x(0)=0, y(0)=1],
       [v(0)=0.1,theta(0)=-Pi/2, x(0)=0, y(0)=1],
       [v(0)=0.8,theta(0)=0, x(0)=0, y(0)=1]],
      theta=-Pi..3*Pi, v=0..2.2,x=-1..10, y=-1..3,
      linecolor=[black,red, green,violet],
     numpoints=300, obsrange=false,
        scene=[x,y]);
```



How do I stop it when y<= 0??? Want a function defined in one way for y>0, another for y<=0.

```
> yphug:=R-> [diff(theta(t),t) = (v(t)^2 - cos(theta(t)))/(v(t)),
              diff(v(t),t)     = -sin(theta(t))-R*v(t)^2
              diff(x(t),t)     = v(t)*cos(theta(t)),
              diff(y(t),t)     = v(t)*sin(theta(t))]:
```