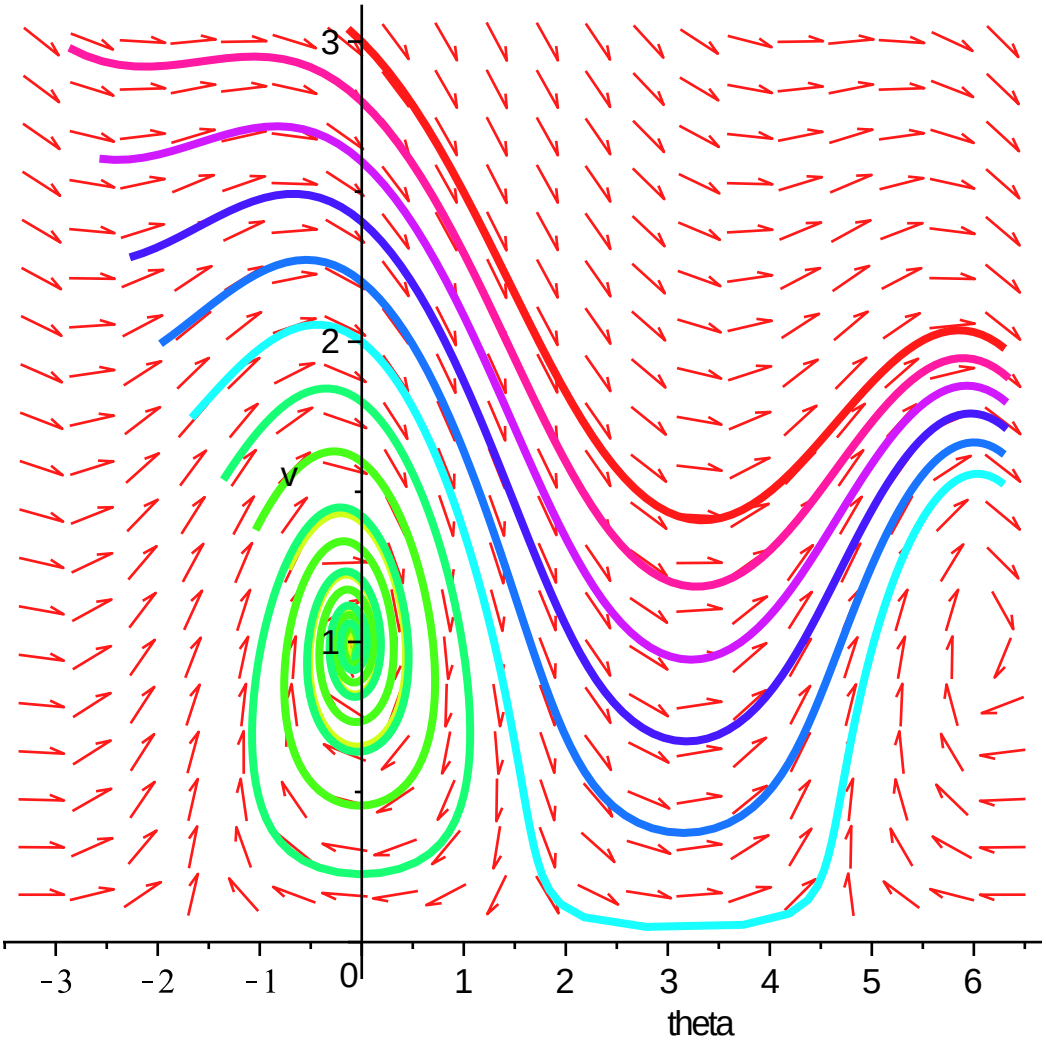
```
> with(DEtools):
  phug:=[ D(theta)(t) = v(t) - cos(theta(t))/v(t),
          D(v)(t)     = -sin(theta(t)) -R*v(t)^2];
```

$$phug := \left[D(\theta)(t) = v(t) - \frac{\cos(\theta(t))}{v(t)}, D(v)(t) = -\sin(\theta(t)) - 0.1\, v(t)^2 \right] \quad (1)$$

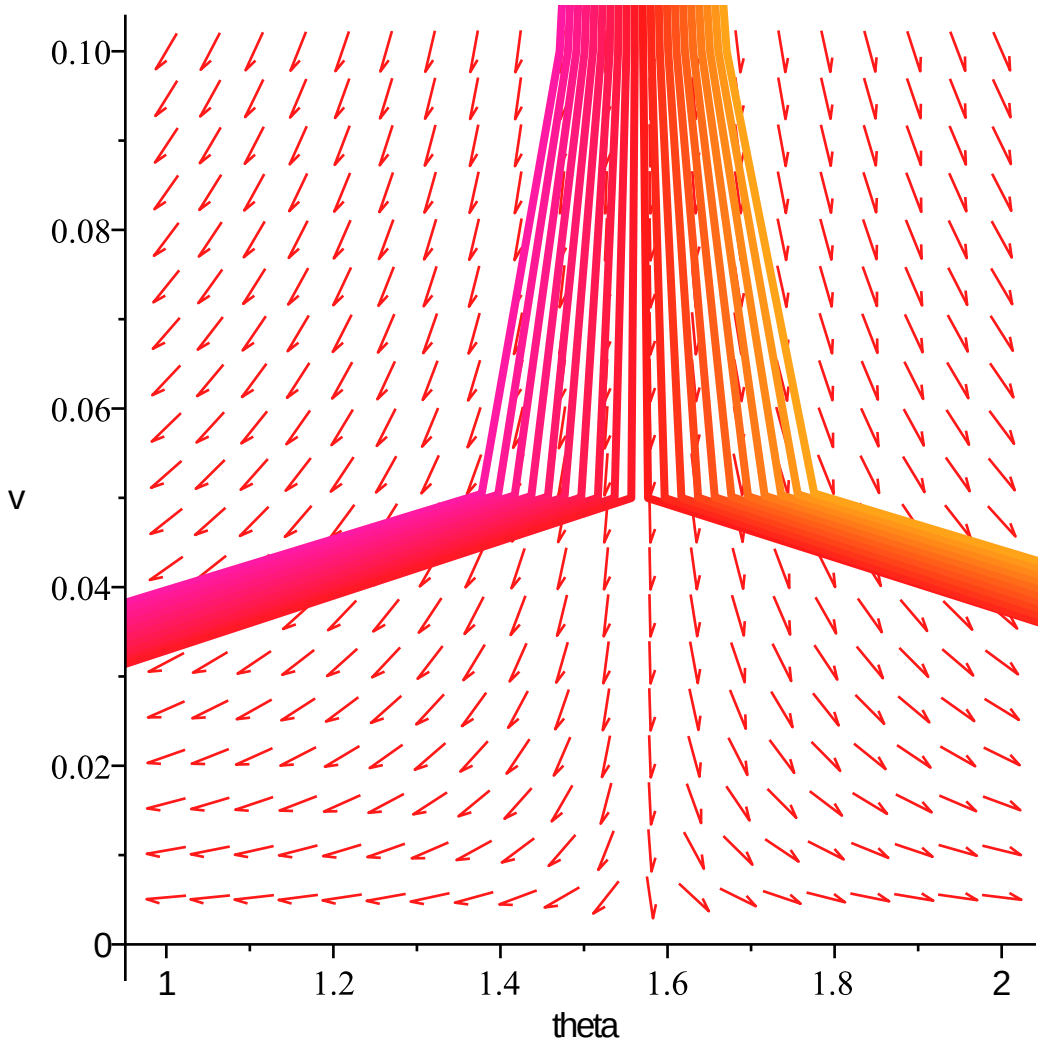
```
> R:=0.3;
  stuff:=[theta(t), v(t)], t=-1..20,
    theta=-Pi..2*Pi, v=0..3,
    [seq([theta(0)=0, v(0)=i],i=1..3,0.2)],
    linecolor=[seq(COLOR(HUE,i),i=0..1,.1)], stepsize=0.05:
  DEplot(phug, stuff, scene=[theta,v]);
```

$$R := 0.3$$

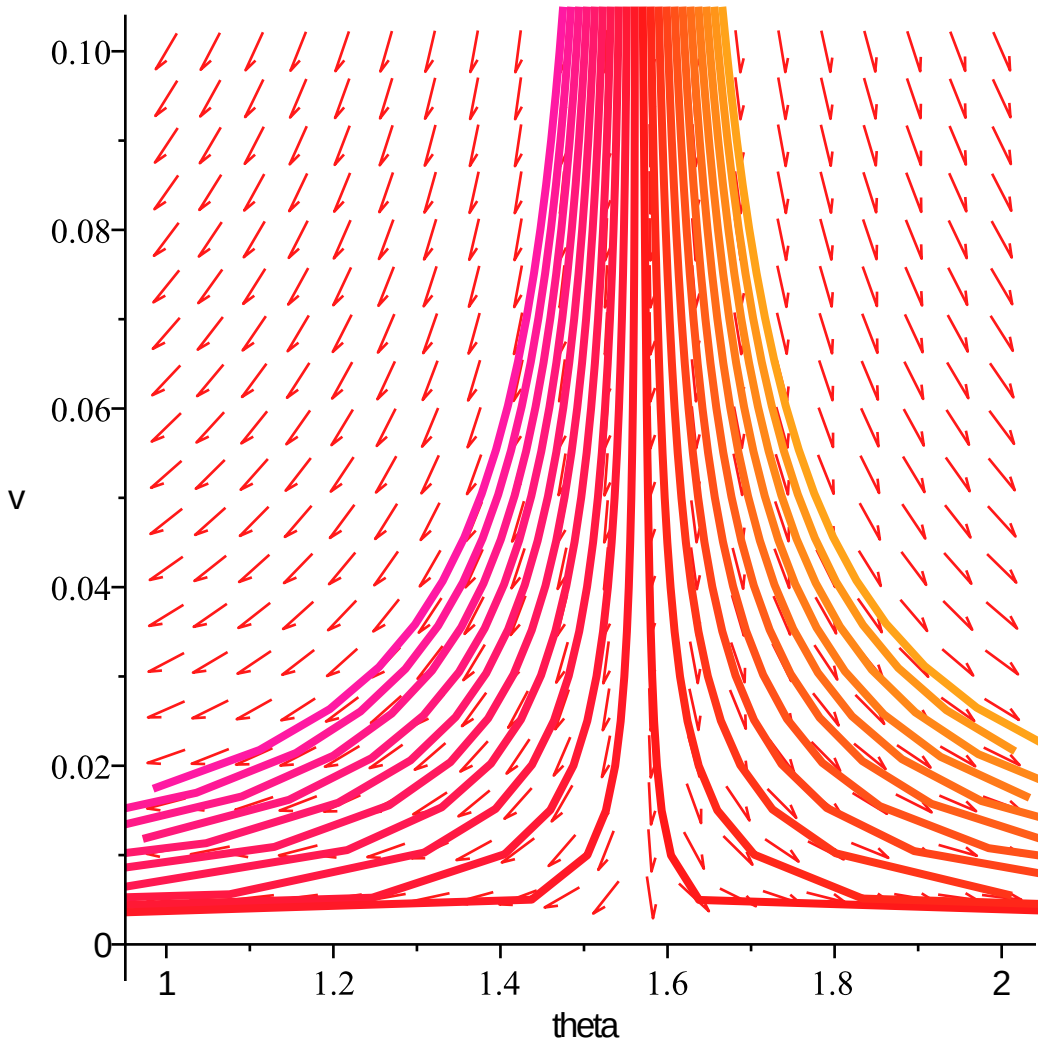



```
> R:=0.3;
  stuff:=[theta(t), v(t)], t=-1..20,
    theta=1..2, v=0..0.1,
    [seq([theta(0)=Pi/2+i, v(0)=0.1],i=-0.1..0.1,0.01)],
    linecolor=[seq(COLOR(HUE,i),i=-0.1..0.1,0.01)], stepsize=0.05:
  DEplot(phug, stuff, scene=[theta,v]);
```

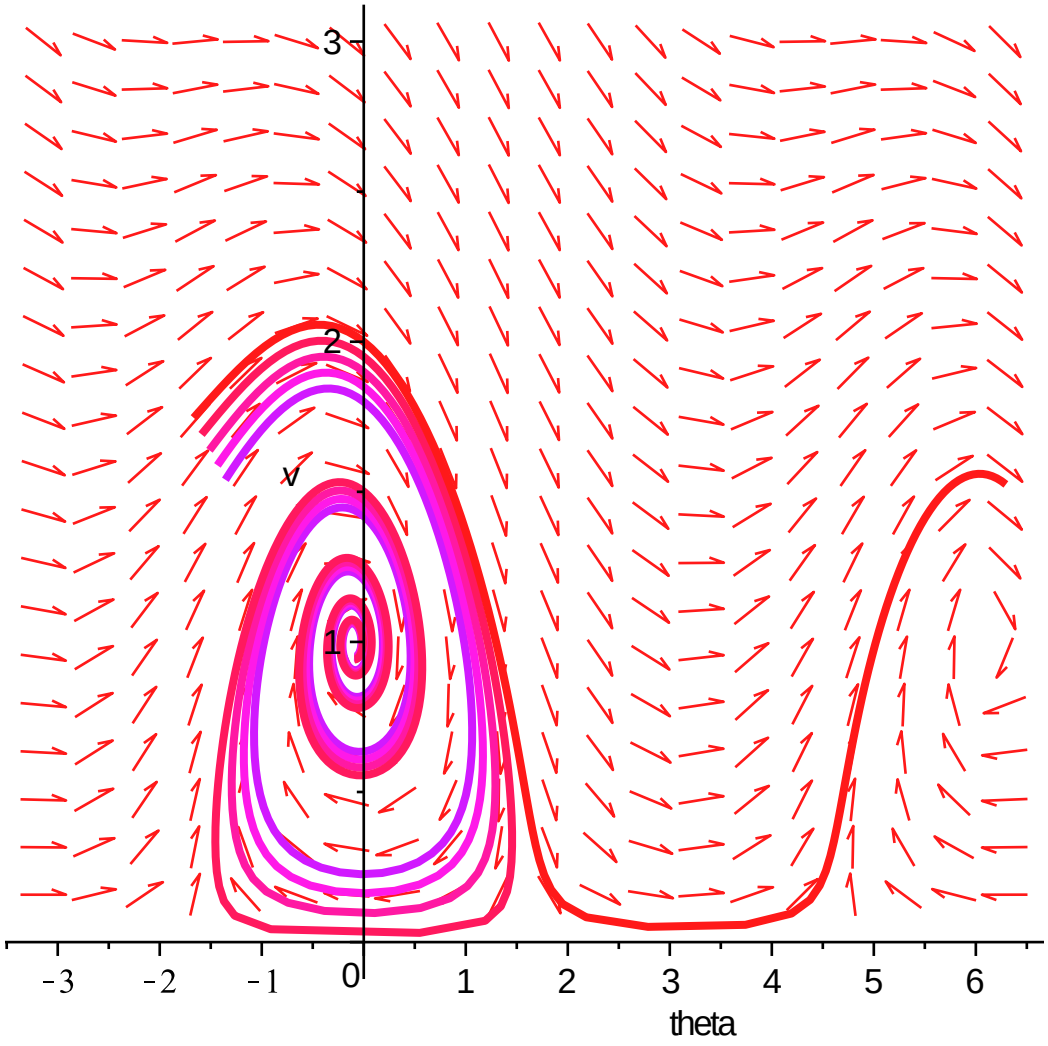
$$R := 0.3$$



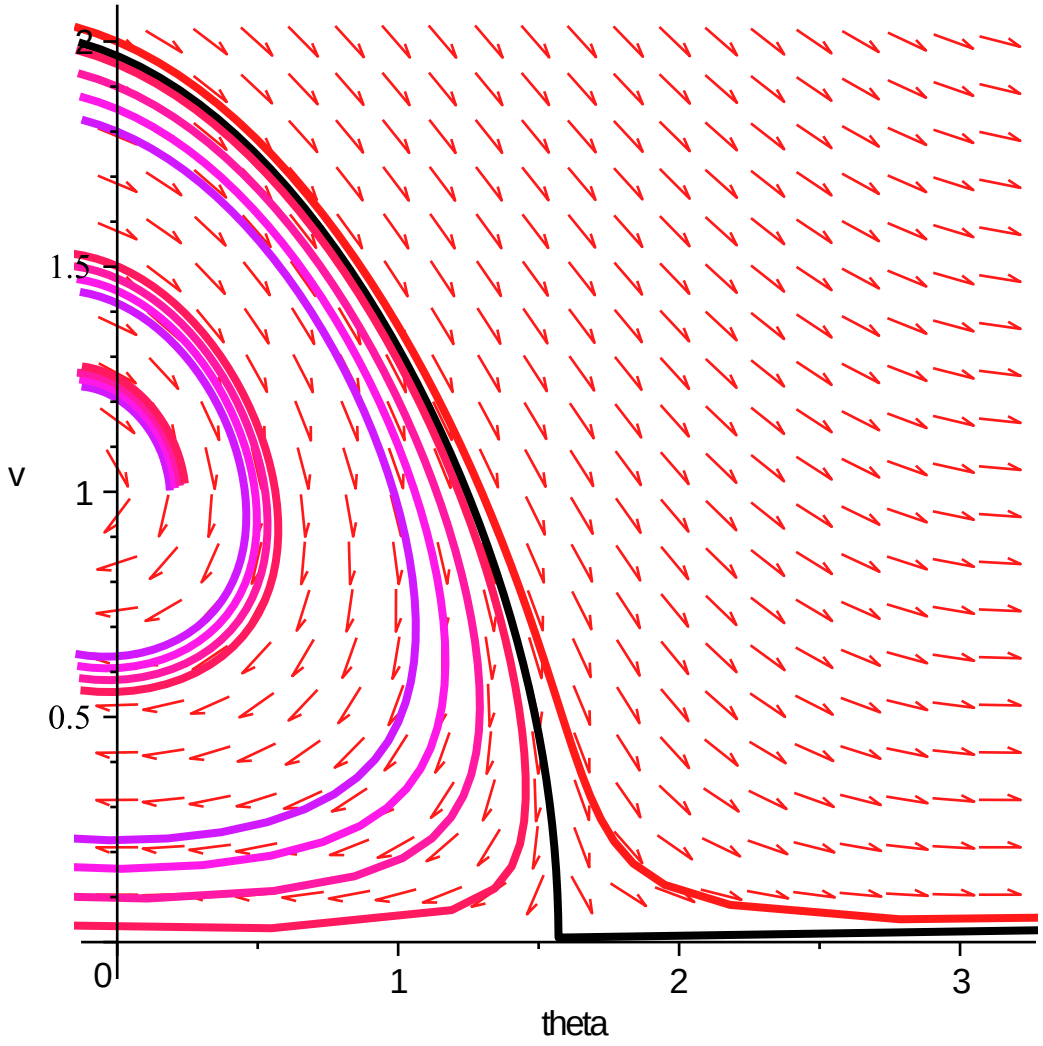

```
> stuff:=[theta(t), v(t)], t=-1..20,
   theta=1..2, v=0..0.1,
   [seq([theta(0)=Pi/2+i, v(0)=0.1],i=-0.1..0.1,0.01)],
   linecolor=[seq(COLOR(HUE,i),i=-0.1..0.1,0.01)], stepsize=0.005:
  DEplot(phug, stuff, scene=[theta,v]);
```



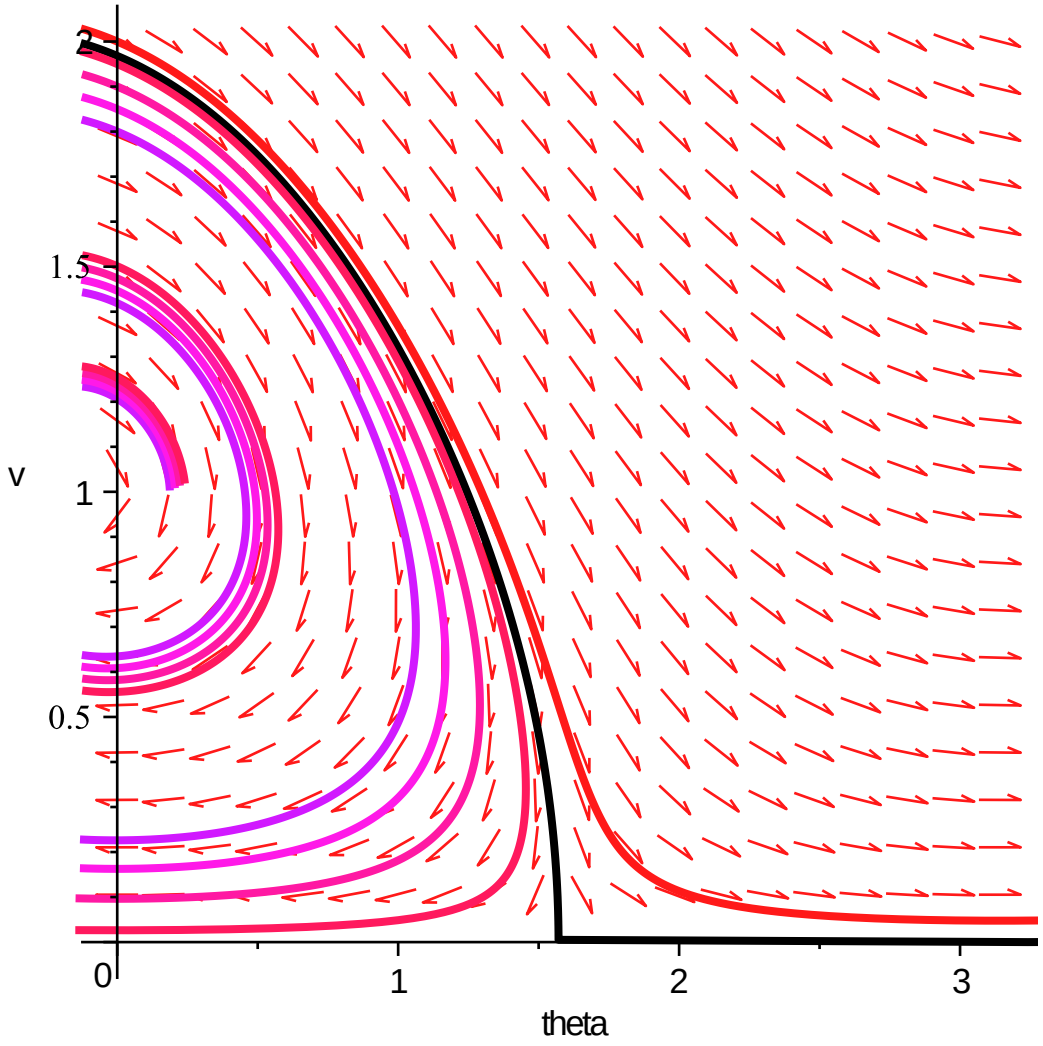

```
> stuff:=[theta(t), v(t)], t=-1..20,
    theta=-Pi..2*Pi, v=0..3,
    [seq([theta(0)=0, v(0)=i],i=1.8..2,0.05)],
   linecolor=[seq(COLOR(HUE,i),i=1.8..2,0.05)], stepsize=0.05:
  DEplot(phug, stuff, scene=[theta,v]);
```




```
> stuff:=[theta(t), v(t)], t=-5..10,
   theta=-0..Pi, v=0..2,
   [[theta(0)=Pi/2, v(0)=0.01],seq([theta(0)=0, v(0)=i],i=1.8..2,
  0.05)],
   linecolor=[black,seq(COLOR(HUE,i),i=1.8..2,0.05)], obsrange=
  false,stepsize=0.05:
  DEplot(phug, stuff, scene=[theta,v]);
```

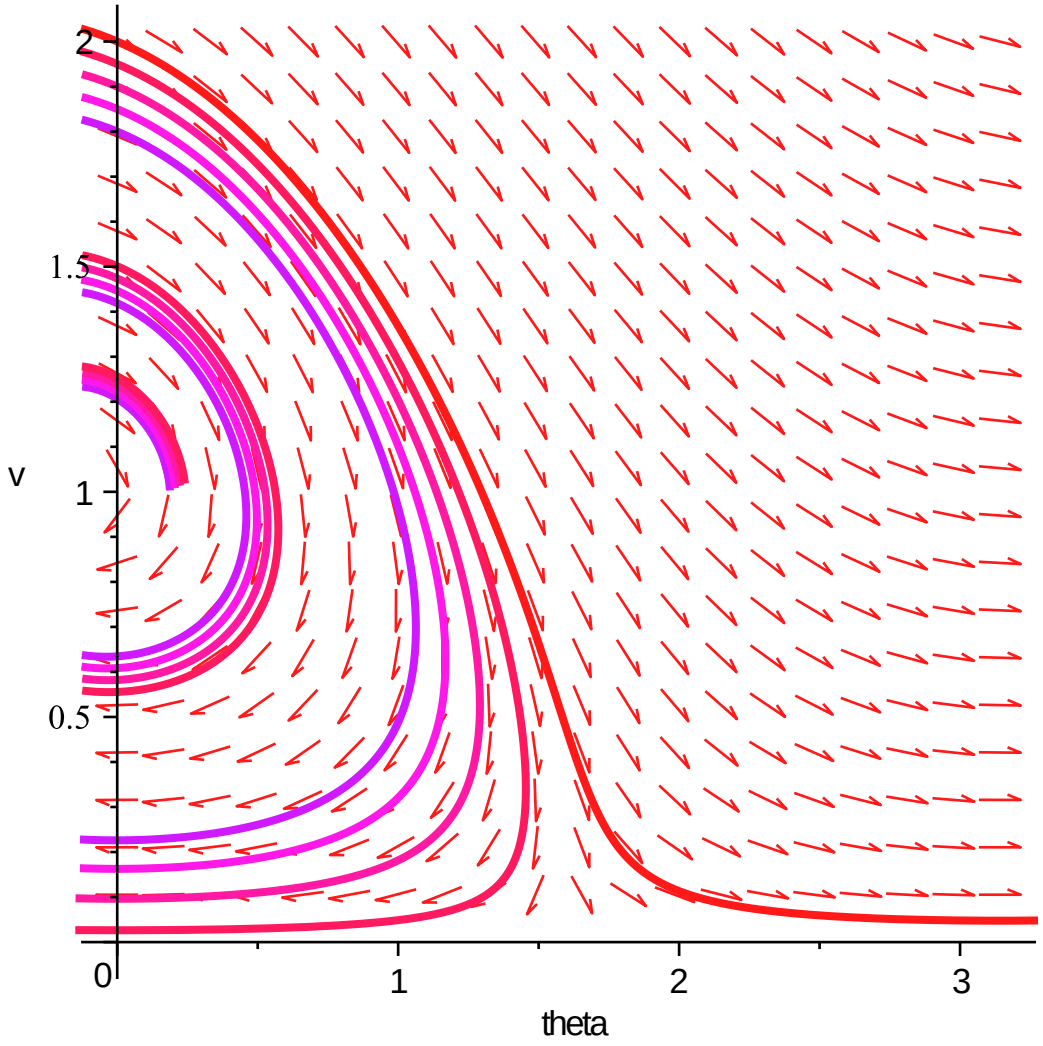



```
> stuff:=[theta(t), v(t)], t=-5..10,
   theta=-0..Pi, v=0..2,
   [[theta(0)=Pi/2, v(0)=0.01],seq([theta(0)=0, v(0)=i],i=1.8..2,
  0.05)],
   linecolor=[black,seq(COLOR(HUE,i),i=1.8..2,0.05)], obsrange=
  false,stepsize=0.005:
  DEplot(phug, stuff, scene=[theta,v]);
```

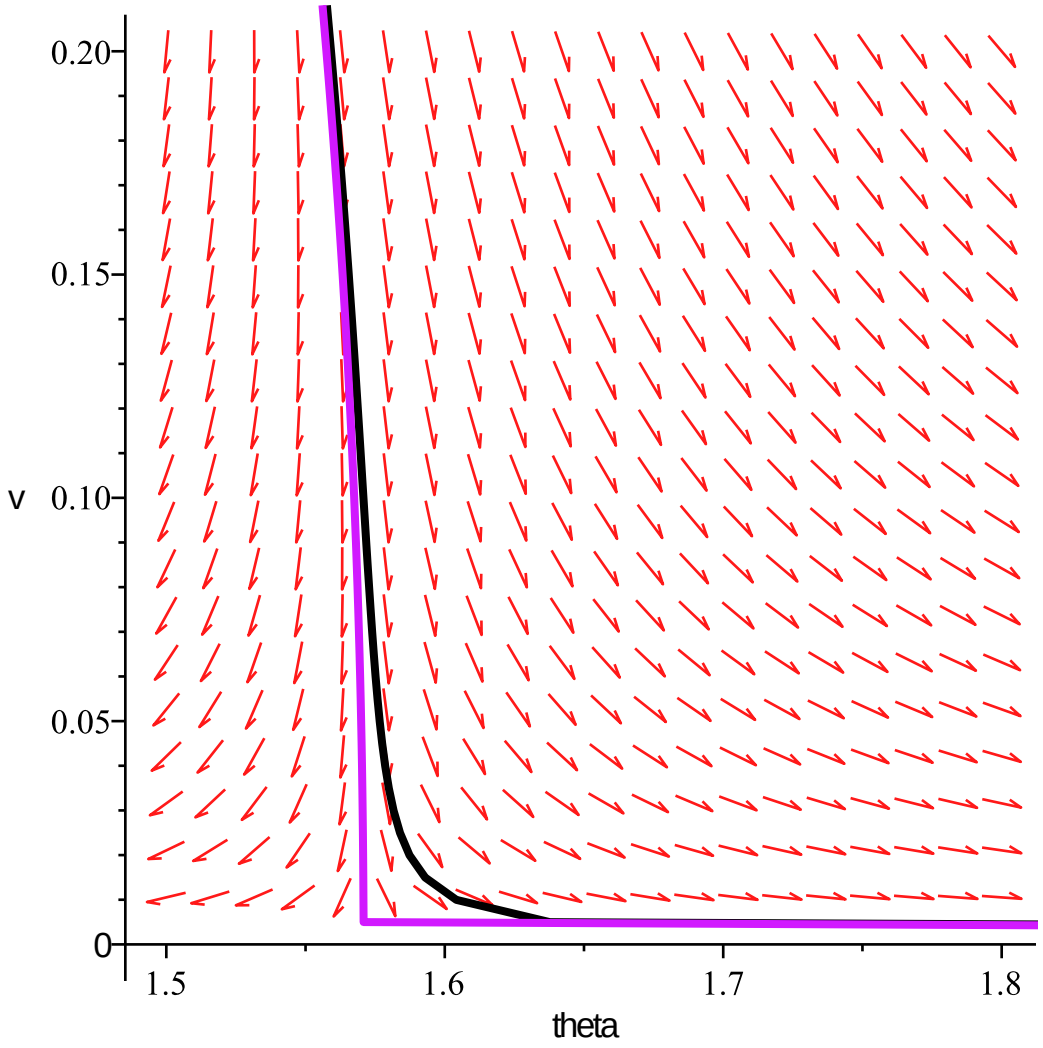



```
> stuff:=[theta(t), v(t)], t=-5..10,
    theta=-0..Pi, v=0..2,
    [[theta(0)=Pi/2, v(0)=0],seq([theta(0)=0, v(0)=i],i=1.8..2,
  0.05)],
    linecolor=[black,seq(COLOR(HUE,i),i=1.8..2,0.05)], obsrange=
  false,stepsize=0.005:
  DEplot(phug, stuff, scene=[theta,v]);
```

Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution past the initial point, problem may be complex, initially singular or improperly set up




```
> stuff:=[theta(t), v(t)], t=-5..10,
   theta=1.5..1.8, v=0..0.2,
   [[theta(0)=Pi/2, v(0)=0.1],[theta(0)=Pi/2,v(0)=0.01]],
   linecolor=[black,seq(COLOR(HUE,i),i=1.8..2,0.05)], obsrange=
  false,stepsize=0.005:
  DEplot(phug, stuff, scene=[theta,v]);
```

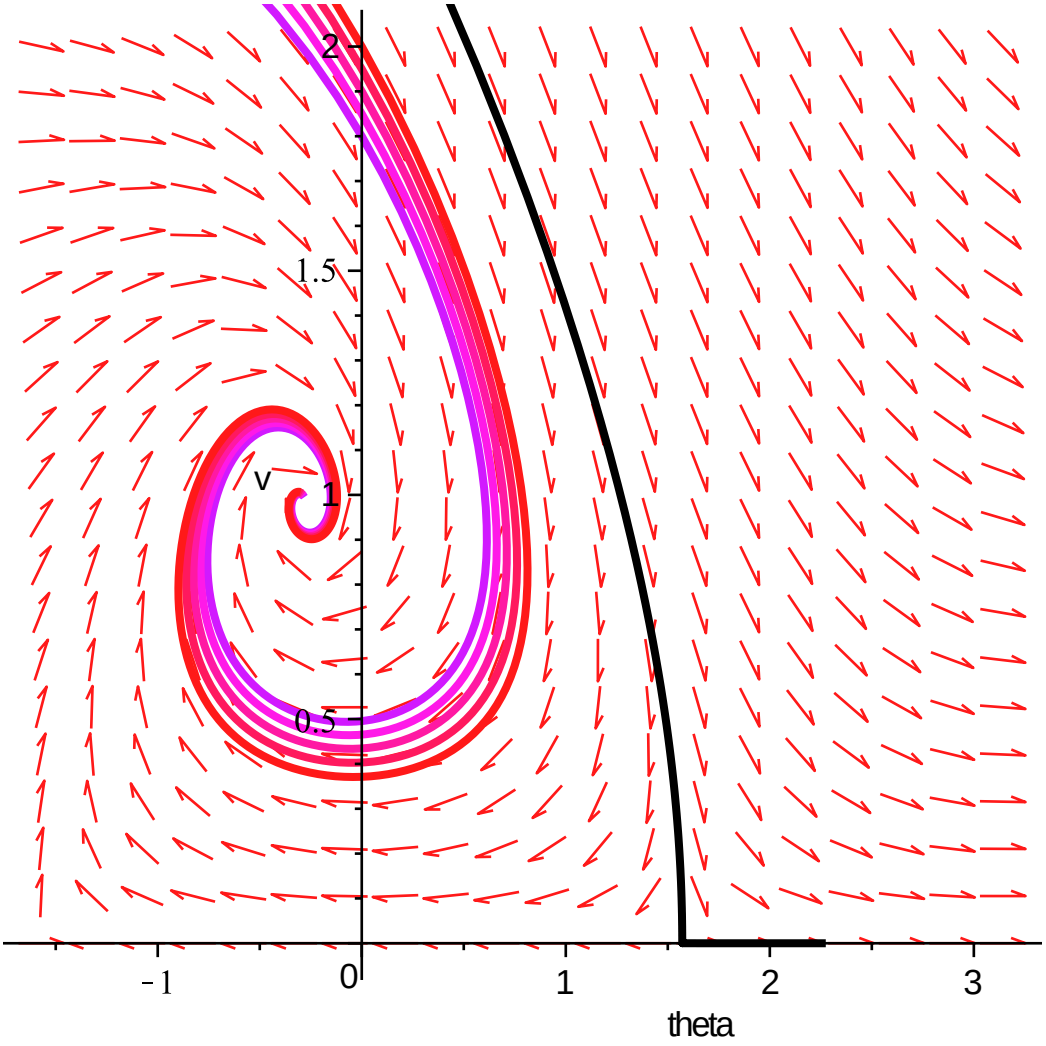



```
> vphug:=[ D(theta)(t) = v(t)^2 - cos(theta(t)),
         D(v)(t)     = (-sin(theta(t)) -R*v(t)^2)*v(t)];
```

$$vphug := \left[D(\theta)(t) = v(t)^2 - \cos(\theta(t)),\ D(v)(t) = \left(-\sin(\theta(t)) - 0.3\, v(t)^2\right) v(t) \right] \quad (2)$$

```
> stuff:=[theta(t), v(t)], t=-5..10,
    theta=-Pi/2..Pi, v=0..2,
    [[theta(0)=Pi/2, v(0)=0.01],seq([theta(0)=0, v(0)=i],i=1.8..2,
  0.05)],
    linecolor=[black,seq(COLOR(HUE,i),i=1.8..2,0.05)], obsrange=
  false,stepsize=0.05:
  DEplot(vphug, stuff, scene=[theta,v]);
Warning, plot may be incomplete, the following errors(s) were
issued:
   cannot evaluate the solution further left of -.69973011,
probably a singularity
Warning, plot may be incomplete, the following errors(s) were
issued:
   cannot evaluate the solution further left of -.64930540,
probably a singularity
Warning, plot may be incomplete, the following errors(s) were
issued:
   cannot evaluate the solution further left of -.60461454,
probably a singularity
```

```
Warning, plot may be incomplete, the following errors(s) were
issued:
   cannot evaluate the solution further left of -.56476163,
probably a singularity
Warning, plot may be incomplete, the following errors(s) were
issued:
   cannot evaluate the solution further left of -.52902775,
probably a singularity
```




```
> F:=(theta,v)-> [ v^2-cos(theta), v*(-sin(theta)-S*v^2)];
```

$$F := (\theta, v) \rightarrow \left[v^2 - \cos(\theta),\ v\left(-\sin(\theta) - S\,v^2\right)\right] \tag{3}$$

```
> with(VectorCalculus):
> Jacobian(F(theta,v), [theta,v]);
```

$$\begin{bmatrix} \sin(\theta) & 2\,v \\ -v\cos(\theta) & -\sin(\theta) - 3\,S\,v^2 \end{bmatrix} \tag{4}$$

```
> eval(%, {theta=Pi/2, v=0});
```

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{5}$$

```
> Jacobian(F(theta,v), [theta,v]):
```

```
eval(%, {theta=-Pi/2, v=0});
```

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \tag{6}$$

Want plane to stop when y(t)=0.

```
> xphug:=[ D(theta)(t) = v(t) - cos(theta(t))/v(t),
         D(v)(t)     = -sin(theta(t)) -R*v(t)^2,
         D(x)(t)     = v(t)*cos(theta),
         D(y)(t)     = v(t)*sin(theta)];
```

$$xphug := \left[\mathrm{D}(\theta)(t) = v(t) - \frac{\cos(\theta(t))}{v(t)}, \mathrm{D}(v)(t) = -\sin(\theta(t)) - 0.3\, v(t)^2, \mathrm{D}(x)(t) \right. \tag{7}$$

$$\left. = v(t)\cos(\theta), \mathrm{D}(y)(t) = v(t)\sin(\theta) \right]$$

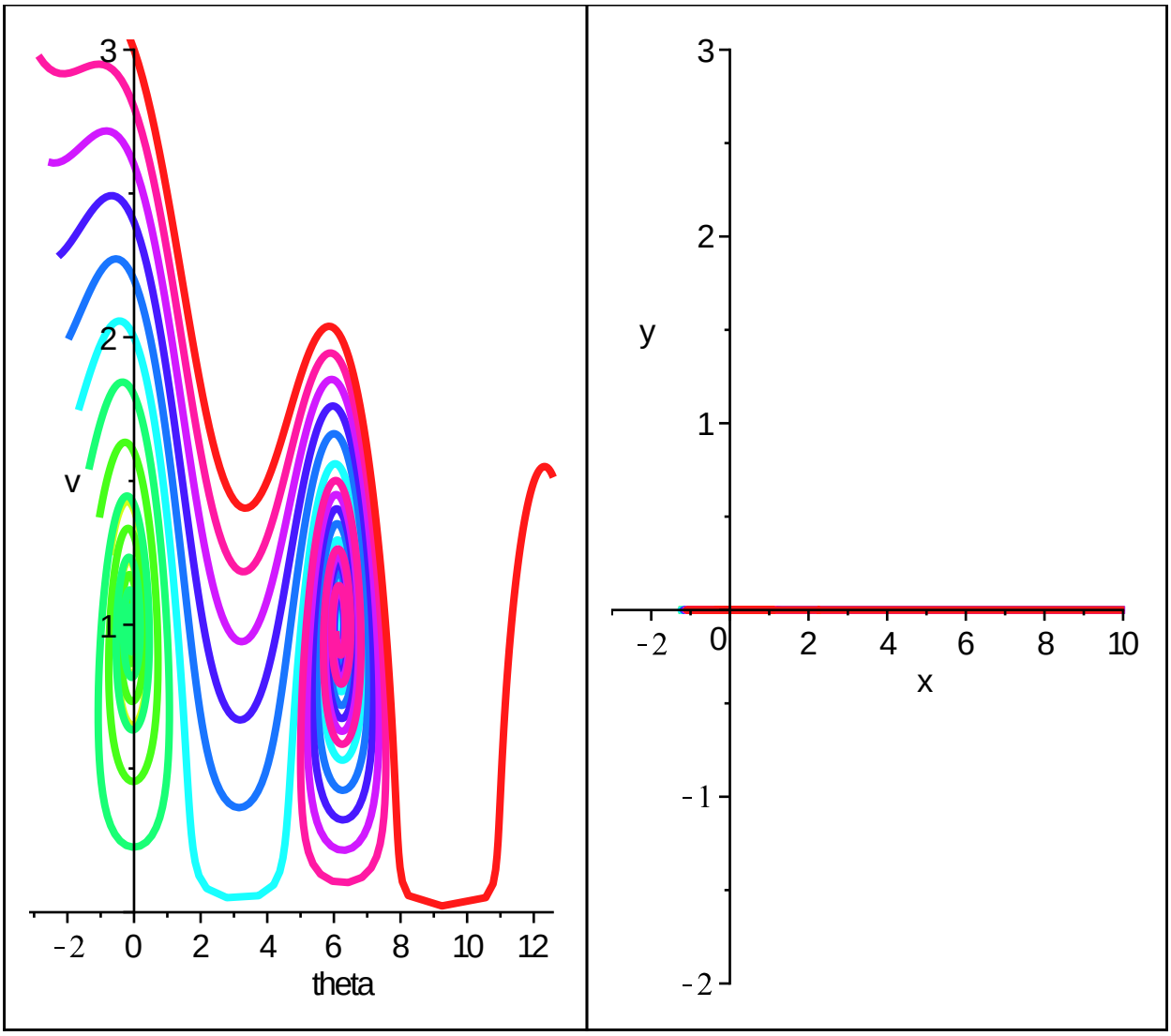
```
> crashy:=[ D(theta)(t) = v(t) - cos(theta(t))/v(t),
         D(v)(t)     = -sin(theta(t)) -R*v(t)^2,
         D(x)(t)     = v(t)*cos(theta(t)),
         D(y)(t)     = piecewise( y(t)>0, v(t)*sin(theta(t)), 0)];
```

$$crashy := \left[\mathrm{D}(\theta)(t) = v(t) - \frac{\cos(\theta(t))}{v(t)}, \mathrm{D}(v)(t) = -\sin(\theta(t)) - 0.1\, v(t)^2, \mathrm{D}(x)(t) \right. \tag{8}$$

$$\left. = v(t)\cos(\theta(t)), \mathrm{D}(y)(t) = \begin{cases} v(t)\sin(\theta(t)) & 0 < y(t) \\ 0 & otherwise \end{cases} \right]$$

```
> with(plots):
> R:=1;
  stuff:=[theta(t), v(t), x(t), y(t)], t=-1..20,
    theta=-Pi..4*Pi, v=0..3, x=-3..10, y=-2..3,
    [seq([theta(0)=0, v(0)=i, x(0)=0, y(0)=0],i=1..3,0.2)],
    linecolor=[seq(COLOR(HUE,i),i=0..1,.1)], stepsize=0.05:
  display( array( [ DEplot(crashy, stuff, scene=[theta,v]),
                    DEplot(crashy, stuff, scene=[x,y]) ]));
```

$$R := 1$$




```
> crashy:=[ D(theta)(t) = piecewise( y(t)>0,v(t) - cos(theta(t))/v
  (t),0),
          D(v)(t)    = piecewise( y(t)>0,-sin(theta(t)) -R*v(t)^2,
  0),
          D(x)(t)    = piecewise( y(t)>0,v(t)*cos(theta(t)),0),
          D(y)(t)    = piecewise( y(t)>0, v(t)*sin(theta(t)),0)];
```

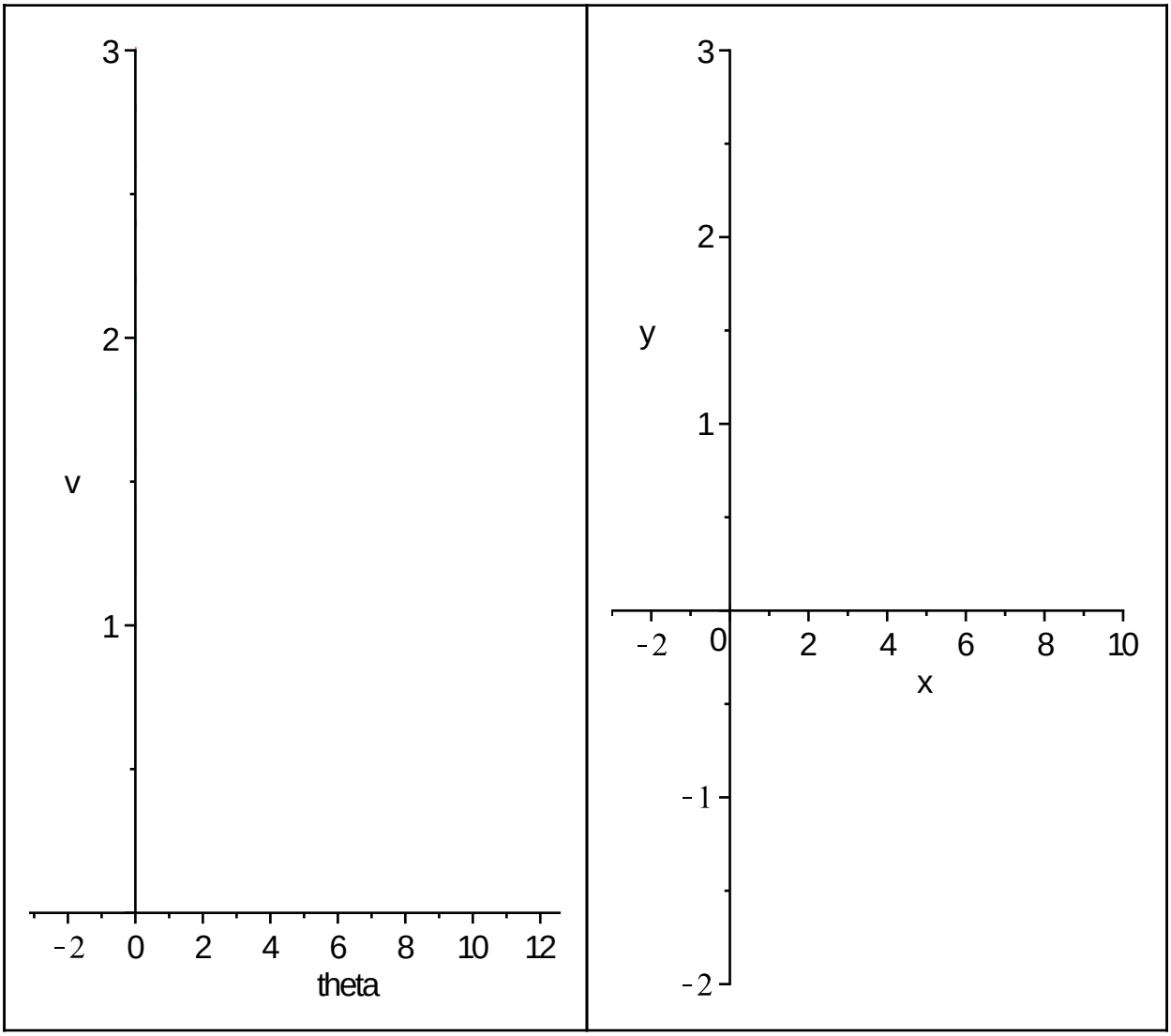
$$crashy := \left[\mathrm{D}(\theta)(t) = \begin{cases} v(t) - \dfrac{\cos(\theta(t))}{v(t)} & 0 < y(t) \\ 0 & otherwise \end{cases}, \mathrm{D}(v)(t) = \right. \tag{9}$$

$$\begin{cases} -\sin(\theta(t)) - 0.5\, v(t)^2 & 0 < y(t) \\ 0 & otherwise \end{cases}, \mathrm{D}(x)(t) = \begin{cases} v(t)\cos(\theta(t)) & 0 < y(t) \\ 0 & otherwise \end{cases},$$

$$\left. \mathrm{D}(y)(t) = \begin{cases} v(t)\sin(\theta(t)) & 0 < y(t) \\ 0 & otherwise \end{cases} \right]$$

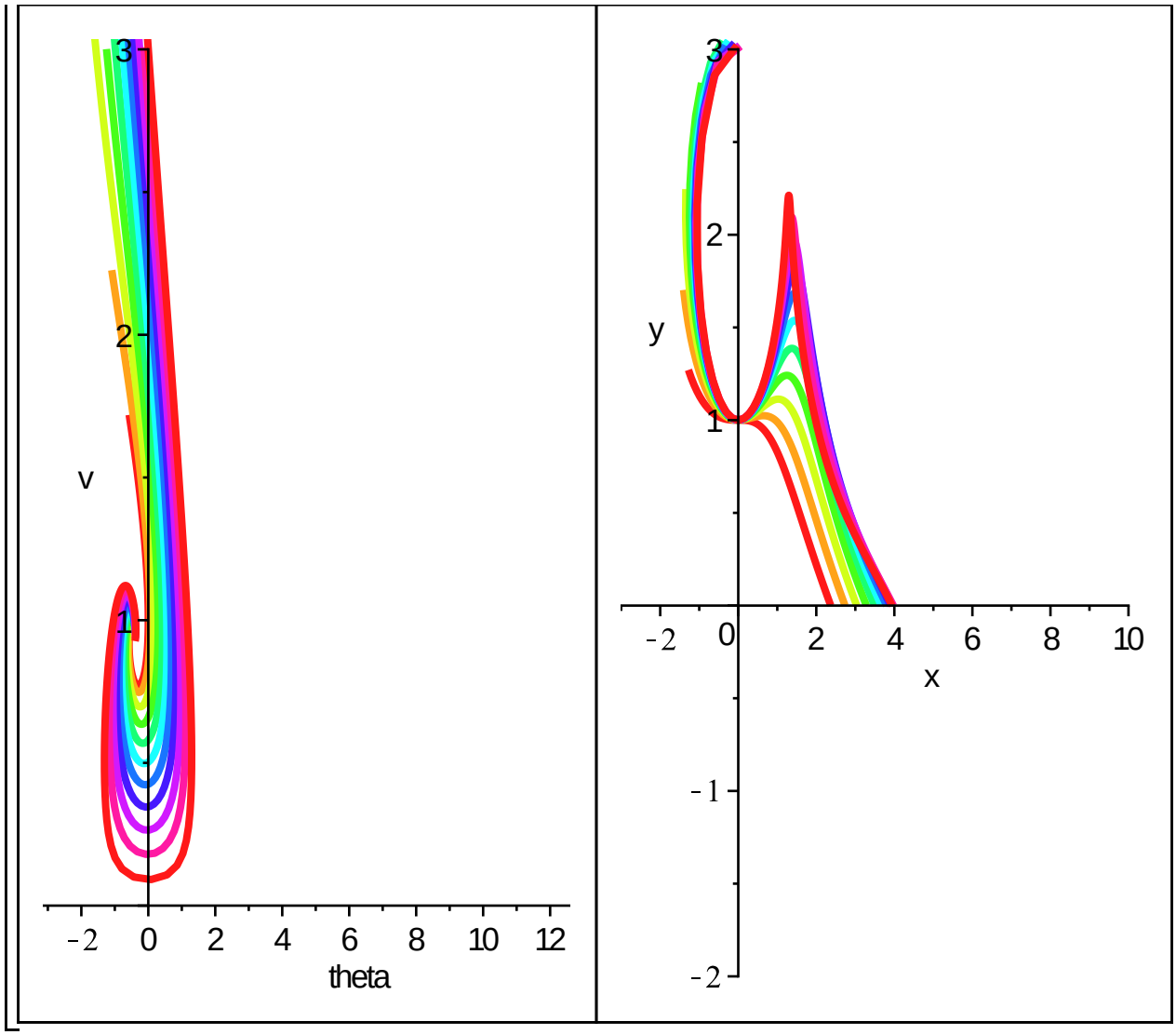
```
> R:=1;
  stuff:=[theta(t), v(t), x(t), y(t)], t=-1..20,
    theta=-Pi..4*Pi, v=0..3, x=-3..10, y=-2..3,
    [seq([theta(0)=0, v(0)=i, x(0)=0, y(0)=0],i=1..3,0.2)],
    linecolor=[seq(COLOR(HUE,i),i=0..1,.1)], stepsize=0.05:
  display( array( [ DEplot(crashy, stuff, scene=[theta,v]),
                    DEplot(crashy, stuff, scene=[x,y]) ]));
```

$$R := 1$$




```
> R:=0.5;
  stuff:=[theta(t), v(t), x(t), y(t)], t=-1..20,
    theta=-Pi..4*Pi, v=0..3, x=-3..10, y=-2..3,
    [seq([theta(0)=0, v(0)=i, x(0)=0, y(0)=1],i=1..3,0.2)],
    linecolor=[seq(COLOR(HUE,i),i=0..1,.1)], stepsize=0.05:
  display( array( [ DEplot(crashy, stuff, scene=[theta,v]),
                    DEplot(crashy, stuff, scene=[x,y]) ]));
```

$$R := 0.5$$


3
2
v
1
-2
0
2
4
6
8
10
12
theta
3
2
y
1
-2
0
2
4
6
8
10
x
-1
-2