```
> with(DEtools):
```

```
> R:='R';
```

$$R := R \quad (1)$$

```
> phug:=[ D(theta)(t) = v(t) - cos(theta(t))/v(t),
          D(v)(t)     = -sin(theta(t)) -R*v(t)^2];
```

$$phug := \left[\mathrm{D}(\theta)(t) = v(t) - \frac{\cos(\theta(t))}{v(t)}, \mathrm{D}(v)(t) = -\sin(\theta(t)) - R\, v(t)^2 \right] \quad (2)$$

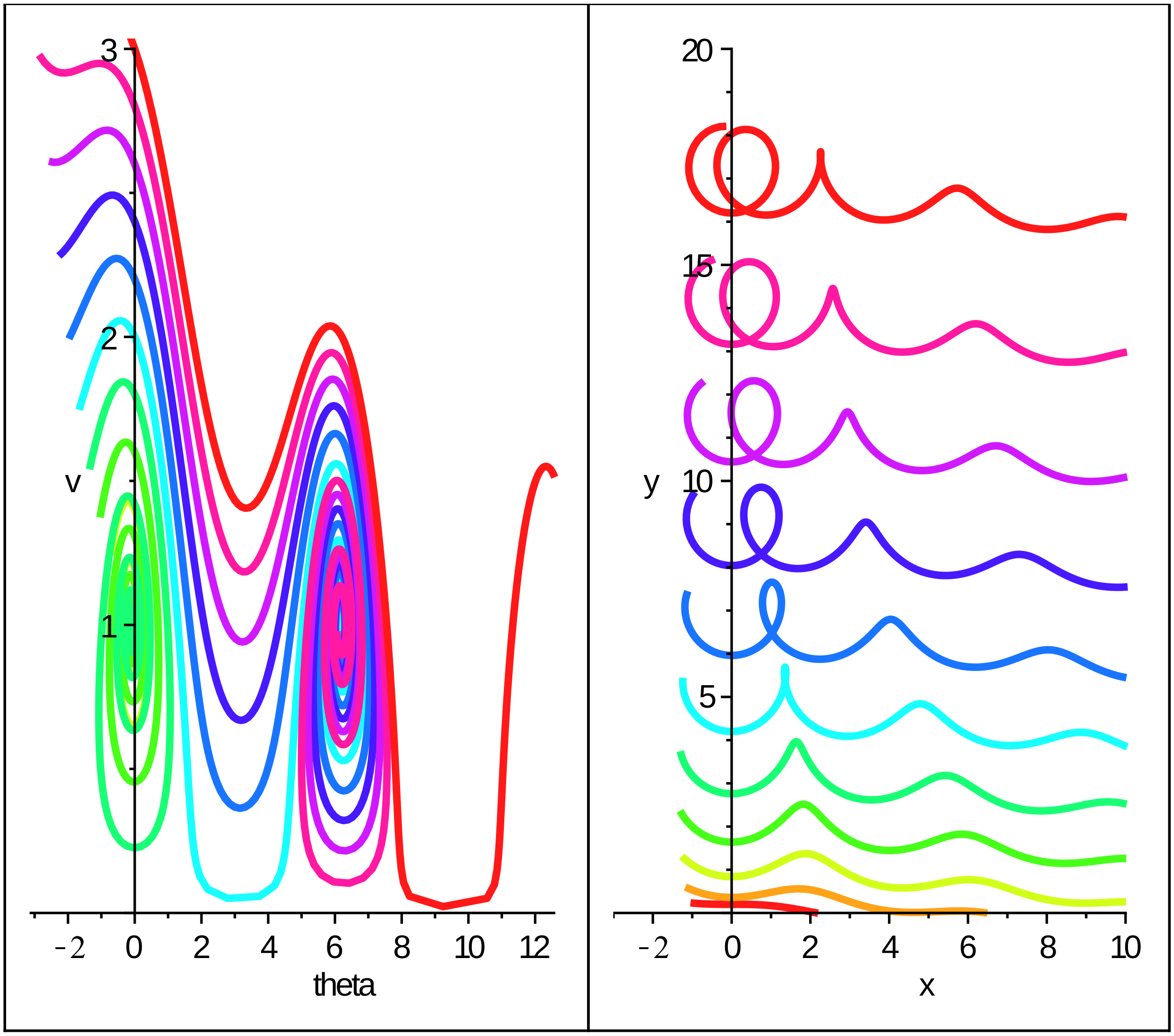
```
> xphug:=[ D(theta)(t) = v(t) - cos(theta(t))/v(t),
           D(v)(t)     = -sin(theta(t)) -R*v(t)^2,
           D(x)(t)     = v(t)*cos(theta(t)),
           D(y)(t)     = v(t)*sin(theta(t))];
```

$$xphug := \left[\mathrm{D}(\theta)(t) = v(t) - \frac{\cos(\theta(t))}{v(t)}, \mathrm{D}(v)(t) = -\sin(\theta(t)) - R\, v(t)^2, \mathrm{D}(x)(t) = v(t) \cos(\theta(t)), \mathrm{D}(y)(t) = v(t) \sin(\theta(t)) \right] \quad (3)$$

```
> with(plots):
```

```
> R:=0.1;
  stuff:=[theta(t), v(t), x(t), y(t)], t=-1..20,
    theta=-Pi..4*Pi, v=0..3, x=-3..10, y=0..20,
    [seq([theta(0)=0, v(0)=i, x(0)=0, y(0)=4*(i-1)^2+.2],i=1..3,
  0.2)],
    linecolor=[seq(COLOR(HUE,i),i=0..1,.1)], stepsize=0.05:
  display( array( [ DEplot(xphug, stuff, scene=[theta,v]),
                    DEplot(xphug, stuff, scene=[x,y]) ]));
```

$$R := 0.1$$




```
> R:='R';
```

$$R := R \tag{4}$$

```
> F:=(theta,v) -> [ v-cos(theta)/v, -sin(theta) - S*v^2];
```

$$F := (\theta, v) \rightarrow \left[v - \frac{\cos(\theta)}{v}, -\sin(\theta) - S\,v^2 \right] \tag{5}$$

```
> solve(F(theta,v)=[0,0]);
```

Error, invalid input: solve expects its 1st argument, eqs, to be of type {`and`, `not`, `or`, algebraic, relation(algebraic), ({set, list})({`and`, `not`, `or`, algebraic, relation(algebraic)})}, but received [v-cos(theta)/v, -sin(theta)-S*v^2] = [0, 0]

```
> F(theta,v)=[0,0];
```

$$\left[v - \frac{\cos(\theta)}{v}, -\sin(\theta) - S\,v^2 \right] = [0, 0] \tag{6}$$

```
> F(theta,v)[1]=0, F(theta,v)[2]=0;
```

$$v - \frac{\cos(\theta)}{v} = 0, -\sin(\theta) - S\,v^2 = 0 \tag{7}$$

```
> solve( {F(theta,v)[1]=0, F(theta,v)[2]=0}, [theta,v]);
```

$$[[\theta = \arctan(-RootOf(-1 + (S^2+1)\ \_Z^2)\ S, RootOf(-1 + (S^2+1)\ \_Z^2)), v = RootOf(\tag{8}$$

$-RootOf(-1+(S^2+1)\_Z^2)+\_Z^2)]]$

```
> convert(%,radical);
```

$$\left[\left[\theta = \arctan\left(-\sqrt{\frac{1}{S^2+1}}\,S, \sqrt{\frac{1}{S^2+1}}\right), v = \left(\frac{1}{S^2+1}\right)^{1/4}\right]\right] \quad \textbf{(9)}$$

```
> fix:= S-> [arctan(-sqrt(1/(S^2+1))*S, sqrt(1/(S^2+1))), sqrt(sqrt
  (1/(1+S^2)))];
```

$$fix := S \rightarrow \left[\arctan\left(VectorCalculus:\text{-}\grave{}\text{-}\grave{}\left(\sqrt{1\,\frac{1}{S^2+1}}\,S\right), \sqrt{1\,\frac{1}{S^2+1}}\right), \sqrt{\sqrt{1\,\frac{1}{1+S^2}}}\right] \quad \textbf{(10)}$$

```
> fix(0.1);
```

$$[-0.09966865249, 0.9975155088] \quad \textbf{(11)}$$

```
> fix(0);
```

$$[0, 1] \quad \textbf{(12)}$$

```
> fix(2);
```

$$\left[-\arctan(2), \frac{1}{5}\,5^{3/4}\right] \quad \textbf{(13)}$$

```
> S:='S';
```

$$S := S \quad \textbf{(14)}$$

```
> with(VectorCalculus):
> Jacobian(F(theta,v), [theta,v]);
```

$$\begin{bmatrix} \frac{\sin(\theta)}{v} & 1+\frac{\cos(\theta)}{v^2} \\ -\cos(\theta) & -2\,S\,v \end{bmatrix} \quad \textbf{(15)}$$

```
> Jack:=unapply(%,(theta,v,S));
```

$$Jack := (\theta, v, S) \rightarrow rtable\left(1..2, 1..2, \left\{(1,1) = \frac{\sin(\theta)}{v}, (1,2) = 1+\frac{\cos(\theta)}{v^2}, (2,1) = -\cos(\theta), (2,2) = -2\,S\,v\right\}, datatype = anything, subtype = Matrix, storage = rectangular, order = Fortran\_order\right) \quad \textbf{(16)}$$

```
> Jack(theta,v,0);
```

$$\begin{bmatrix} \frac{\sin(\theta)}{v} & 1+\frac{\cos(\theta)}{v^2} \\ -\cos(\theta) & 0 \end{bmatrix} \quad \textbf{(17)}$$

```
> Jack(0,1,0);
```

$$\begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix} \quad \textbf{(18)}$$

```
> with(LinearAlgebra):
> Eigenvalues(Jack(0,1,0));
```

$$\begin{bmatrix} I\sqrt{2} \\ -I\sqrt{2} \end{bmatrix} \tag{19}$$

```
> fix(0.1);\
```

$$[-0.09966865249, 0.9975155088] \tag{20}$$

```
> Jack(-0.09917726108, .9975155088, 0.1);
```

$$\begin{bmatrix} -0.09926136830 & 2.000049018 \\ -0.9950859653 & -0.1995031018 \end{bmatrix} \tag{21}$$

```
> Eigenvalues(%);
```

$$\begin{bmatrix} -0.149382235050000 + 1.40986120112587\,I \\ -0.149382235050000 - 1.40986120112587\,I \end{bmatrix} \tag{22}$$

```
> fix(2);
```

$$\left[-\arctan(2), \frac{1}{5}\,5^{3/4}\right] \tag{23}$$

```
> Jack(op(fix(2)),2);
```

$$\begin{bmatrix} -\frac{2}{5}\,5^{3/4} & 2 \\ -\frac{1}{5}\sqrt{5} & -\frac{4}{5}\,5^{3/4} \end{bmatrix} \tag{24}$$

```
> Eigenvalues(%);
```

$$\begin{bmatrix} -\frac{3}{5}\,5^{3/4} - \frac{1}{5}\,I\,5^{3/4} \\ -\frac{3}{5}\,5^{3/4} + \frac{1}{5}\,I\,5^{3/4} \end{bmatrix} \tag{25}$$

```
> evalf(%);
```

$$\begin{bmatrix} -2.006220915 - 0.6687403050\,I \\ -2.006220915 + 0.6687403050\,I \end{bmatrix} \tag{26}$$

```
> Jack(op(fix(3)),3);
```

$$\begin{bmatrix} -\frac{3}{10}\,10^{3/4} & 2 \\ -\frac{1}{10}\sqrt{10} & -\frac{3}{5}\,10^{3/4} \end{bmatrix} \tag{27}$$

```
> Eigenvalues(%);
```

$$\begin{bmatrix} -\frac{1}{2}\,10^{3/4} \\ -\frac{2}{5}\,10^{3/4} \end{bmatrix} \tag{28}$$

```
> evalf(%);
```

$$\begin{bmatrix} -2.811706626 \\ -2.249365301 \end{bmatrix} \quad (29)$$

```
> Jack(op(fix(S)), S);
```

$$\begin{bmatrix} -\frac{\left(\frac{1}{S^2+1}\right)^{1/4} S}{\sqrt{\frac{S^2}{S^2+1}+\frac{1}{S^2+1}}} & 1+\frac{1}{\sqrt{\frac{S^2}{S^2+1}+\frac{1}{S^2+1}}} \\ -\frac{\sqrt{\frac{1}{S^2+1}}}{\sqrt{\frac{S^2}{S^2+1}+\frac{1}{S^2+1}}} & -2\,S\left(\frac{1}{S^2+1}\right)^{1/4} \end{bmatrix} \quad (30)$$

```
> simplify(%);
```

$$\begin{bmatrix} -S\left(\frac{1}{S^2+1}\right)^{1/4} & 2 \\ -\sqrt{\frac{1}{S^2+1}} & -2\,S\left(\frac{1}{S^2+1}\right)^{1/4} \end{bmatrix} \quad (31)$$

```
> Mat:= unapply(%,S);
```

$$Mat := S \rightarrow rtable\left(1..2, 1..2, \left\{(1,1) = -S\left(\frac{1}{S^2+1}\right)^{1/4}, (1,2) = 2, (2,1) = -\sqrt{\frac{1}{S^2+1}}, (2,2) = -2\,S\left(\frac{1}{S^2+1}\right)^{1/4}\right\}, datatype = anything, subtype = Matrix, storage = rectangular, order = Fortran\_order\right) \quad (32)$$

```
> Mat(2);
```

$$\begin{bmatrix} -\frac{2}{5}\,5^{3/4} & 2 \\ -\frac{1}{5}\sqrt{5} & -\frac{4}{5}\,5^{3/4} \end{bmatrix} \quad (33)$$

```
> Mat(3);
```

$$\begin{bmatrix} -\frac{3}{10}\,10^{3/4} & 2 \\ -\frac{1}{10}\sqrt{10} & -\frac{3}{5}\,10^{3/4} \end{bmatrix} \quad (34)$$

```
> CharacteristicPolynomial(Mat(S), lambda);
```

$$\lambda^2 + 3\,S\left(\frac{1}{S^2+1}\right)^{1/4}\lambda + 2\sqrt{\frac{1}{S^2+1}} + 2\,S^2\sqrt{\frac{1}{S^2+1}} \quad (35)$$

```
> ?characteristic
> Trace(Mat(S));
```

$$-3\,S\left(\frac{1}{S^2+1}\right)^{1/4} \quad (36)$$

```
> Determinant(Mat(S));
```

$$2\sqrt{\frac{1}{S^2+1}}+2\,S^2\sqrt{\frac{1}{S^2+1}} \quad (37)$$

```
> Trace(Mat(S))^2 = 4*Determinant(Mat(S));
```

$$9\,S^2\sqrt{\frac{1}{S^2+1}}=8\sqrt{\frac{1}{S^2+1}}+8\,S^2\sqrt{\frac{1}{S^2+1}} \quad (38)$$

```
> solve(%,S);
```

$$2\sqrt{2},\ -2\sqrt{2} \quad (39)$$

```
> Mat(2*sqrt(2));
```

$$\begin{bmatrix} -\frac{2}{9}\sqrt{2}\,9^{3/4} & 2 \\ -\frac{1}{9}\sqrt{9} & -\frac{4}{9}\sqrt{2}\,9^{3/4} \end{bmatrix} \quad (40)$$

```
> Eigenvalues(%);
```

$$\begin{bmatrix} -\sqrt{2}\sqrt{3} \\ -\sqrt{2}\sqrt{3} \end{bmatrix} \quad (41)$$

```
> Eigenvectors(Mat(3));
```

$$\begin{bmatrix} -\frac{1}{2}\,10^{3/4} \\ -\frac{2}{5}\,10^{3/4} \end{bmatrix},\ \begin{bmatrix} -10^{1/4} & -2\,10^{1/4} \\ 1 & 1 \end{bmatrix} \quad (42)$$

```
> evalf(%);
```

$$\begin{bmatrix} -2.811706626 \\ -2.249365301 \end{bmatrix},\ \begin{bmatrix} -1.778279410 & -3.556558820 \\ 1. & 1. \end{bmatrix} \quad (43)$$