

Math 331, Fall 2002: Problems 17-20

17. (*expires 10/28*) Consider the differential equation  $\dot{\mathbf{z}}(t) = \mathbf{F}(\mathbf{z}(t))$ , where the vector  $\mathbf{z}(t) = (x(t), y(t))$  and the field  $\mathbf{F}(x, y) = (-y, x - y)$ . Plot a few solutions. What happens to them when  $t \rightarrow +\infty$ ? Give a “Maple-proof” that this is a general fact for every solution. [A “Maple-proof” is an argument that is rigorous once we accept Maple results as incontrovertibly true.]

18. (*expires 10/28*) (No Maple.) For the equation  $\dot{\mathbf{z}} = \mathbf{F}(\mathbf{z})$ ,  $\mathbf{z} = (x, y)$ , with the vector field

$$\mathbf{F}(x, y) = \langle -x(x^4 + y^4) - y, x - y(x^4 + y^4) \rangle,$$

prove that the origin is an attractor in the future, i.e., every solution verifies

$$\lim_{t \rightarrow +\infty} \mathbf{z}(t) = 0.$$

[You can ask around how to do this, but then you have to show clearly that you have understood it.]

19. (*expires 10/28*) For the system of differential equations of prob. #23,

$$\begin{cases} \dot{x} = x^2 + y, \\ \dot{y} = x(y^2 - 1), \end{cases}$$

find the eigenvalues and eigenvectors of the Jacobian at the fixed points. [This is a give-away if you have done #16.]

20. (*expires 10/28*) Consider the equations of the glider with no drag term ( $R = 0$ ). Use `dsolve, type=numeric` to solve them numerically with initial conditions  $\theta(0) = 0$ ,  $v(0) = 0.8$ . Then solve exactly the linearized system around the fixed point  $(\theta_0, v_0) = (0, 1)$ , with the same initial conditions. Graph the two functions for  $0 \leq t \leq 5$ , and give a good estimate of their maximum difference. What happens if we take a larger  $t$ -range?