Math 331, Fall 2002: Problems 1-6

NOTE: *Each exercise is worth 10 points and can be turned in at any time before its "expiration date". At the end of the semester, I will expect you to have turned in at least 2/5 of the exercises assigned. If you do more, I will pick your best grades. If you do less, the missing grades will be counted as zeros. Altogether, these will count the same as one project.*

1. (*expires 9/23*) Use `Maple` to write $x^5 - 2x^4 - 10x^3 + 20x^2 - 16x + 32$ as a product of *exact* linear factors. By exact, I mean you should leave any non-rational factors expressed as radicals; do not approximate terms like $\sqrt{3}$ as 1.73205, etc.

2. (*expires 9/23*) Draw a graph showing both $\cos(x)$ and its fifth Taylor polynomial (that is, $1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4$) for x between -4 and 4. What degree of Taylor polynomial seems to be needed to get good agreement in this range" *Hint: use a variation of the command* `convert(taylor(cos(x),x,5),polynom)` *to make this work. Think of a suitable way to demonstrate that the approximation you have taken is "good"– what is a good definition of "good" here?*

3. (*expires 9/30*) Consider the planar curve γ defined by $x^2y^3 + y^2 + y - 2e^x = 0$. Using **only** `Maple`, find the slope of the tangent line to the curve at $(0, 1)$. Then plot the curve and the tangent line on the same graph.
Hint: you might want to use `implicitplot` *from the library* `plots`*. You might find* `implicitdiff` *helpful, too.*

4. (*expires 9/30*) Plot the function $f(x) = 2\sin x - x^3 - 1/5$, for $x \in [-4, 4]$. Find all the zeros of the function with an accuracy of 20 decimal digits. *Hint: See* `Digits`, `fsolve`.

5. (*expires 9/30*) Define a `Maple` function g that, given a positive integer k yields the sum of the first k primes. What is k such that $g(k) \leq 100,000$ but $g(k+1) > 100,000$? You might find `sum` and `ithprime` helpful.

6. (*expires 9/30*) Use the Taylor expansion of $\arctan x$ near the point $x = 1/\sqrt{3}$ to compute the value of π to 30 places. How many terms are needed to compute the value to 50 places?