Second Midterm Exam MAT 310, Spring 2005

Name: **ID#:** **Rec:**

problem	1	2	3	4	5	Total
possible	25	25	25	25	25	100
score						

Directions: There are 5 problems on six pages (including this one) in this exam. Make sure that you have them all. Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it.

Do any four problems. Cross out the one you don't want graded.

You may use any bound books or a calculator to do this exam. Using extra papers, notes, computers, or discussions with friends (or enemies) is not permitted. If you wish to make use of a time machine to look at the solutions, you may do so provided you admit such usage below, and then allow me to use it to retroactively change the questions.

Failure to admit such usage of a time machine will be grounds for charges of academic dishonesty (as is more "ordinary" methods of cheating).

Do any four problems. Cross out the one you don't want graded.

1. *(25 points)* Let $T \in \mathcal{L}(\mathbb{R}^2, \mathbb{R}^2)$ satisfy $T\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ and $T\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

a) Write a matrix which represents T in the standard basis.

b) Write a matrix which represents T in the basis $\left\{\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix}\right\}$.

Do any four problems. Cross out the one you don't want graded.

2.*(25 points)* Let T be the transformation whose matrix in the standard basis is $\begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 3 \\ 0 & 0 & 1 \end{pmatrix}$

a) What is the characteristic polynomial of T?

b) What is the minimal polynomial of T?

c) Is T similar over $\mathbb{R}$ to a diagonal matrix A? While you must justify your answer, you don't have to exhibit A if your answer is yes. An answer of "yes" or "no" without justification will receive at most one point of the nine possible.

Do any four problems. Cross out the one you don't want graded.

3. *(25 points)* Let $\mathcal{P}$ be the infinite dimensional vector space of all polynomials with real coefficients, and let $f \in \mathcal{L}(\mathcal{P}, \mathbb{R})$ be the linear functional given by

$$f(p) = \int_0^1 p(x)\,dx$$

If $D \in \mathcal{L}(\mathcal{P}, \mathcal{P})$ is the differentiation operator (ie, $D(p) = p'$), what is $D^t(f)$?
Here D^t denotes the transpose of D.

Do any four problems. Cross out the one you don't want graded.

4. *(25 points)* Let A be a 3×3 matrix with real entries. Prove that if A is **not** similar over $\mathbb{R}$ to an upper-triangular matrix, then A must be diagonalizable over $\mathbb{C}$.

Do any four problems. Cross out the one you don't want graded.

5. *(25 points)* Let $\mathbb{V}$ be a finite dimensional vector space over $\mathbb{C}$, and let $T \in \mathcal{L}(\mathbb{V}, \mathbb{V})$ be invertible. Prove that if $c \neq 0$ is an eigenvalue of T, then $1/c$ is an eigenvalue of T^{-1}.