MATH 308 Second Midterm

April 13, 2011

Name: ______________________ **ID:** __________________

Question:	1	2	3	4	5	Total
Points:	20	15	15	10	10	70
Score:						

There are 5 problems on 4 pages in this exam (not counting the cover sheet). Make sure that you have them all.

You **may use a calculator** if you wish, provided your calculator does not do calculus. However, it is unlikely to be of much help.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, extra papers, and discussions with friends are not permitted.**

There is a table of Laplace transforms on the next page. Use it, or don't. Applying the inverse Laplace transform to turn annoying algebraic calculations into "simpler" calculus problems probably won't work, but feel free to try. Just don't make me grade your attempts.

You have 80 minutes to do this exam. (more than enough time for me or Yury to complete this exam, but maybe not enough for you. Life is unfair.)

Table of Laplace Transforms

$f(t)$ for $t \geq 0$	$\mathcal{L}[f](s)$
1	$\dfrac{1}{s}$
e^{at}	$\dfrac{1}{s-a}$
t^n	$\dfrac{n!}{s^{n+1}}$ $(n = 0, 1, \ldots)$
$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$ $(n = 0, 1, \ldots)$
$\sin \omega t$	$\dfrac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\dfrac{s}{s^2+\omega^2}$
$t \sin \omega t$	$\dfrac{2\omega s}{(s^2-\omega^2)^2}$
$t \cos \omega t$	$\dfrac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$\delta(t-a)$	e^{-as}
$H_a(t)$	$\dfrac{e^{-as}}{s}$
$H_a(t) f(t-a)$	$e^{-as}\mathcal{L}[f](s)$
$e^{at} f(t)$	$\mathcal{L}[f](s-a)$
$f'(t)$	$s\mathcal{L}[f](s) - f(0)$
$f''(t)$	$s^2\mathcal{L}[f](s) - sf(0) - f'(0)$

Recall that $H_a(t)$ is the Heaviside function, $H_a(t) = \begin{cases} 0 & t < a \\ 1 & t \geq a \end{cases}$

and $\delta(t)$ is the Dirac δ-function[1], with $\delta(t) = 0$ for all $t \neq 0$ and $\displaystyle\int_{-\infty}^{\infty} \delta(t) dx = 1$.

[1] although it is not really a function, but a measure

10 pts. 1. (a) Solve the initial value problem below for $x(t)$ by any method except cheating.

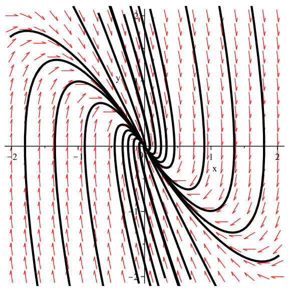
$$x''(t) - x'(t) - 6x(t) = 0, \qquad x(0) = 0, \quad x'(0) = 10$$

10 pts. (b) Find the most general form of $x(t)$ for the inhomogeneous linear equation

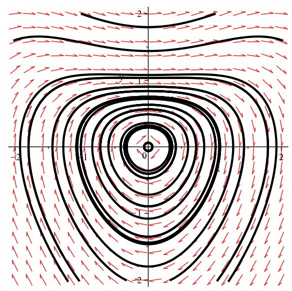
$$x''(t) - x'(t) - 6x(t) = t$$

$\boxed{\text{15 pts.}}$ 2. Below are five second order differential equations labeled (a) through (e), and four phase portraits labeled 1 through 4 with a number of trajectories drawn. On the line following of each of the equations, write the letter of the corresponding phase portrait or the word "none" if the phase portrait is not shown.

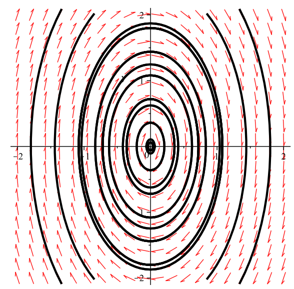
1.



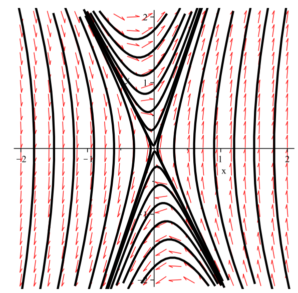
2.



3.



4.



(a) $x'' + 3x = 0$

(a) ______________

(b) $x'' - \sin(x)x' + x = 0$

(b) ______________

(c) $x'' + 4x' + 4x = 0$

(c) ______________

(d) $x'' + x' + 2x = 0$

(d) ______________

(e) $x'' - x' - 6x = 0$

(e) ______________

3. Agent Orange is peacefully relaxing in his spaceship, the Defoliant, completely at rest. Suddenly, an alien battlecruiser appears 5 klicks away and applies a tractor beam which causes the Defoliant to accelerate towards it at a rate of $1 \frac{k}{m^2}$. Fortunately, Agent Orange's countermeasures automatically kick in and are able to counteract the beam's force so that it decreases linearly to zero over the course of one minute. (Unfortunately, Agent Orange forgot to pick up any Tylium last time he was at the store, so his engines won't start and he still drifting towards the aliens.)

If $y(t)$ is the position of the Defoliant at time t, the following differential equation holds (with an appropriate choice of units):

$$y''(t) = \begin{cases} 1-t & 0 \leq t \leq 1 \\ 0 & t < 0 \text{ or } t > 1 \end{cases} \qquad y(0) = y'(0) = 0.$$

$\boxed{\text{10 pts.}}$ (a) Solve for $y(t)$. You may want to use the table of Laplace transforms given earlier.

$\boxed{\text{5 pts.}}$ (b) Agent Orange has a teleporter which takes exactly 10 minutes to prepare for use. If he starts preparing immediately upon sighting the aliens, does he have time to escape? (Justify your answer).

$\boxed{\text{10 pts.}}$ 4. Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$. Calculate e^{At}.

$\boxed{\text{10 pts.}}$ 5. Give the general solution to the system of differential equations

$$\frac{dx}{dt} = x + y \qquad \frac{dy}{dt} = 2y + z \qquad \frac{dz}{dt} = 3z$$