MATH 200, Lec 2   First Midterm

October 13, 2006

Name: _____ **ID:** _____

Question:	1	2	3	4	5	6	Total
Points:	8	8	6	8	9	8	47
Score:							

There are 6 problems in this exam. The pages are printed on both sides. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, extra papers, and discussions with friends are not permitted.** You may use a time machine is to travel ahead and check the answers, provided you let me use it afterwards to travel back in time and change the questions.

You have an hour to complete this exam.

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1. (a) $\boxed{4 \text{ points}}$ Write a statement that is logically equivalent to the one below, but uses no negatives.

   If you didn't do the homework, then you won't pass the exam.

   (b) $\boxed{4 \text{ points}}$ Write the negation of the statement below, using no negatives:

   For every positive real number ε and for every integer x, there is an integer y so that

   $$0 \leq \frac{x}{y} \quad and \quad \frac{x}{y} < \varepsilon$$

2. $\boxed{8 \text{ points}}$ Prove that for any integer n, if n^2 is odd, then n is odd.

3. $\boxed{6 \text{ points}}$ Prove that for any sets A, B, and C, $(A \cap C) - B = (A - B) \cap C$

4. $\boxed{8 \text{ points}}$ Prove that for any positive integer n, $4^n + 5$ is divisible by 3. You might find induction helpful. Recall that $4 = 3 + 1$.

5. Indicate whether each of the following statements is true or false, and justify your answer with a proof. statements:

(a) $\boxed{3 \text{ points}}$ $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, \ x + y > 0$ True   False

(b) $\boxed{3 \text{ points}}$ $\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, \ x + y > 0$ True   False

(c) $\boxed{3 \text{ points}}$ $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, \ xy \geq 0$ True   False

6. Let $f : \mathbb{R} \to \mathbb{R}^2$ be given by $f(x) = (x + 1, x^2 + 1)$.

  (a) $\boxed{4 \text{ points}}$ Is f surjective? Prove or disprove your answer.

  (b) $\boxed{4 \text{ points}}$ Is f injective? Prove or disprove your answer.

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