MATH 200 First Midterm

March 15, 2010

Name: ______________________ **ID:** ________________

Question:	1	2	3	4	5	6	7	Total
Points:	10	8	8	8	8	9	12	63
Score:								

There are 7 problems on 8 pages in this exam (not counting the cover sheet). Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, extra papers, and discussions with friends are not permitted.** If you wish to use your psychic abilities to read my mind for the answers, you may do so. However, remember that I may be deliberately thinking of the *wrong* answers during the test.

You have about 79 minutes and 47 seconds to complete this exam.

When you complete this exam, if there is sufficient time it is strongly recommended that you go back and reexamine your work, both on this exam and in your life up until now, for any errors that you may have made.

4 pts. 1. (a) Write a statement that is logically equivalent to the one below, but uses no negatives.

If an integer is not odd, then it cannot be written as the product of two odd integers.

6 pts. (b) Let $S \subseteq \mathbb{R}$. Write the negation of the statement below, using no negatives:

There exists a positive real number M so that if $x \in S$, then $-M < x < M$.

Give an example of a set of real numbers for which the property above fails to hold.

$\boxed{\text{8 pts.}}$ 2. For any sets A, B, and C, prove that

$$\text{if } A \cap B \subseteq C \text{ and } x \in B, \text{ then } x \notin A - C.$$

Feel free to use Venn diagrams to illustrate your proof, but a proof by Venn diagrams alone will not be given full credit.

$\boxed{\text{8 pts.}}$ 3. Prove that for any real number $x \neq 1$ and any positive integer n,

$$\sum_{j=0}^{n} x^j = \frac{1 - x^{n+1}}{1 - x}.$$

Using induction on n will likely be helpful.

$\boxed{\text{8 pts.}}$ 4. Let $f : X \to Y$ and $g : Y \to Z$ be injective functions. Is $h = g \circ f$ an injective function? Give a proof or counterexample.

$\boxed{\text{8 pts.}}$ 5. Prove that for every pair of integers m and n, if $n - m$ is even, then $n^2 - m$ is also even.

(You may freely use the fact that the sum of two odd numbers is even, that the sum of an odd number and an even number is odd, the product of an odd and an even is even, etc.)

6. Indicate whether each of the following statements is true or false, and justify your answer with a proof. You may use the usual properties of real numbers, including those of inequalities.

$\boxed{\text{3 pts.}}$ (a) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R},\ 0 < y < x$ True False

$\boxed{\text{3 pts.}}$ (b) $\forall x \in \mathbb{R}^+, \exists y \in \mathbb{R},\ 0 < y < x$ True False

$\boxed{\text{3 pts.}}$ (c) $\exists y \in \mathbb{R}^+, \forall x \in \mathbb{R},\ 0 < y < x$ True False

$\boxed{\text{6 pts.}}$ 7. (a) Let $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ be given by $f(x, y) = (x + y^2, 2y)$.
Is f injective? surjective? bijective? Prove or disprove your answer.

$\boxed{\text{6 pts.}}$ (b) Let $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ be given by $f(x, y) = (x + y^2, 2y)$.
Is f injective? surjective? bijective? Prove or disprove your answer.

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