MATH 200, Lec 2 Second Midterm

November 20, 2006

Name: ______________________ **ID:** __________________

Question:	1	2	3	4	5	6	Total
Points:	10	10	10	5	10	10	55
Score:							

There are 6 problems in this exam. The pages are printed on both sides. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, extra papers, and discussions with friends are not permitted.** You may contact the psychic friends network telepathically for help, but I don't think Miss Cleo or Dionne Warwick know much math.

You have an hour to complete this exam.

Axioms and Definitions for Geometry

UNDEFINED TERMS. The plane is our universe of discourse, and points and lines are subsets of the plane. A line is a set of points with properties as defined by the axioms. The distance between any two points A and B is a number denoted by $|AB|$, again with properties as specified by the axioms.

INCIDENCE AXIOM.

1. For any two distinct points, there is a unique line that contains these two points.
2. Every line contains at least two distinct points.
3. For any line, there exists a point not on this line.

DEFINITION. Two lines l and m are said to be transverse if they are distinct ($l \neq m$) and have at least one point in common. Two lines are parallel if they are not transverse.

THE PARALLEL AXIOM. For any line l and a point P not on l, there exists a unique line containing P and parallel to l.

THE RULER AXIOM. Let l be any line. Then there is a bijection $f : l \to \mathbb{R}$ such that, for any two points A, B on l, the distance between A and B, $|AB|$, is given by $|f(A) - f(B)|$. This bijection f is called a coordinate system on l.

DEFINITION. If A, B, and C are points on a line l, we say B is between A and C if there is a coordinate system f on l for which $f(A) < f(B) < f(C)$. The set of all points on l that are between between A and C is called the line segment $\overline{AB}$.

DEFINITION. Let A, B and C be three distinct points on a line l. We say that A and C are on opposite sides of B if B is between A and C. If A and B are not on opposite sides of C, we say A and B are on the same side of C.

DEFINITION. If l is a line and V and A are distinct points on l, we define the ray $\overrightarrow{VA}$ to be all of the points on l that are on the same side of V as A.

1. 10 points Prove that there is no rational number whose square is 3.
You may assume that if a is an integer, a^2 is divisible by 3 if and only if a is divisible by 3.

2. (a) 5 points Show that if A and B are disjoint denumerable sets, then $A \cup B$ is also denumerable.

(b) 5 points Show that if X is an uncountable set and $A \subseteq X$ is denumerable, then the complement of A in X (that is, $X - A$) must be uncountable.
You may use the first part of this question, even if you couldn't do it

3. Three people decide to get tacos, and the tacqueria serves five kinds of tacos: beef, chicken, pork, fish, and vegetarian. Each person orders exactly one taco.

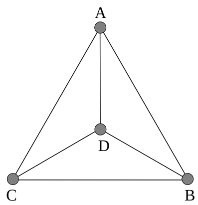
   (a) [5 points] How many choices are possible if we record who selected which dish (as the waiter should)?

   (b) [5 points] How many choices are possible if we forget who ordered which dish (as the chef might)?
   Be careful, this is more complicated than it may seem at first.

4. 5 points What is the coefficent of x^9 in the expansion of $(x+2)^{12}$?

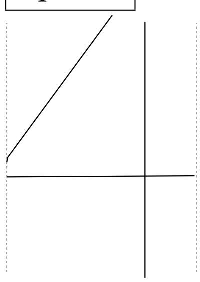
5. **10 points** **Using only the definitions and axioms** on the back of the cover sheet, prove that if ℓ, m, and n are lines so that ℓ is parallel to m, and m is parallel to n, then ℓ is parallel to n.

6. For each of the interpretations of the terms point, line, and distance given below, determine if they are consistent with the axioms given on the back of the cover sheet. If the interpretation is not consistent, state **all** axioms it contradicts, and explain why.



(a) 5 points The plane contains exactly four points, A, B, C, and D. There are six lines: $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{AD}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{BD}$, and $\overleftrightarrow{CD}$, and the distances between points are given by $|AD| = |BD| = |CD| = 1$ and $|AB| = |BC| = |CA| = \sqrt{3}$.

(b) 5 points Points are elements $(x, y) \in \mathbb{R}^2$ with $-1 < x < 1$. A line is the set of points which satisfy $y = mx+b$ where m and b are real numbers (and $-1 < x < 1$); in addition, the points which satisfy $x = a$ where $-1 < a < 1$ are also lines. The distance between two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{\left(\frac{x_1}{x_1^2-1} - \frac{x_2}{x_2^2-1}\right)^2 + (y_1-y_2)^2}$



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