MATH 141 Second Midterm

November 19, 2012

Name: ________________________ **ID:** ___________________

Question:	1	2	3	4	Total
Points:	20	20	20	20	80
Score:					

There are 4 problems on 4 pages in this exam (not counting this cover sheet). Make sure that you have them all.

You may use a calculator if you wish, provided your calculator does not do calculus. However, it is unlikely to be of much help.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, extra papers, and discussions with friends are not permitted.** You may use a time machine to travel forward in time, read the solutions online (or even ask me how to do them), and then return with them. However, if you do so, you must subsequently make the time machine available for my use, so that I may travel back in time and retroactively change the problems beforewards. Any temporal paradoxes created in this way could negatively affect your ultimate grade.

$\boxed{\text{5 pts.}}$ 1. (a) Give a complete and careful definition of the derivative of a function $f(x)$ at the point $x = a$.

$\boxed{\text{5 pts.}}$ (b) Give a complete statement of the Extreme Value Theorem.

$\boxed{\text{5 pts.}}$ (c) Let $f : A \to B$ where A and B are both sets of real numbers. Define what the statement "The function g is the inverse of f" means.

$\boxed{\text{5 pts.}}$ (d) Give a definition of the following statement: "The function $f(x)$ has an essential discontinuity at $x = a$."

2. For each of the functions below, calculate its derivative.

$\boxed{\text{5 pts.}}$ (a) $f(x) = \cos 2x \ln 3x$

$\boxed{\text{5 pts.}}$ (b) $g(x) = |x|^3$

$\boxed{\text{5 pts.}}$ (c) $h(x) = (1 + \sin x)^x$

$\boxed{\text{5 pts.}}$ (d) $a(x) = \arcsin(2x^{1/2})$

$\boxed{\text{20 pts.}}$ 3. Let

$$f(x) = \begin{cases} \sin x & \text{if } x \text{ is rational} \\ x & \text{if } x \text{ is irrational} \end{cases} .$$

Is $f(x)$ differentiable at $x = 0$? If your answer is yes, calculate $f'(0)$. In either case, justify your answer fully.

Hint: don't panic. Use the force (or maybe problem (1a)).

$\boxed{\text{20 pts.}}$ 4. The Fundamental Theorem of Algebra tells us that if $p_n(z)$ is any polynomial of degree n, it can factored into a constant times the product of n linear terms. That is, there are complex numbers $\rho_j, \quad j = 1 \ldots n$ and a constant a so that

$$p_n(z) = a(z - \rho_1)(z - \rho_2)(z - \rho_3) \ldots (z - \rho_n).$$

Use this to show (using induction on the degree n) that the derivative of any polynomial p_n can be written in the form

$$p_n'(z) = p_n(z) \sum_{j=1}^{n} \frac{1}{z - \rho_j}, \qquad (*)$$

provided $p_n(z) \neq 0$.
Hint: you may assume $a = 1$ to make things a little easier, since if $a \neq 1$ we may instead work with the polynomial $q(z) = p(z)/a$.