MAT132, Paper Homework 1

due in recitation on 2/13, 2/14, or 2/15

1. A lemur rancher needs to invest in some high-tech lemur grooming machines. She determines that the machines will depreciate at a rate $f(t) = e^{-at^2}$ (for some constant a), and the cost of keeping them in top running condition is given by another function $g(t) = B\ln(1+t)$, where t is the time that the machines have been running.

   The cost of keeping the machines around (instead of replacing them with new ones) is given by

$$C(t) = \frac{1}{t}\int_0^t (f(x) + g(x))\ dx$$

   Show the critical points of $C(t)$ occur when $C(t) = f(t) + g(t)$ by calculating the derivative of $C(t)$ and setting it to zero.

2. In the problem below, the identities $\cos(\frac{\pi}{2} - x) = \sin(x)$ and $\sin^2(x) + \cos^2(x) = 1$ will be useful.

   (a) Use substitution to show that for *any* continuous function f, $\displaystyle\int_0^{\pi/2} f(\sin x)\ dx = \int_0^{\pi/2} f(\cos x)\ dx$.

   (b) Using part (a) and the other trig identity, calculate $\displaystyle\int_0^{\pi/2} \sin^2(x)dx$ and $\displaystyle\int_0^{\pi/2} \cos^2(x)dx$.