MAT132, Paper Homework 7

1. The hyperbolic sine function $\sinh(x)$ and the hyperbolic cosine $\cosh(x)$ are related as follows:

$$\begin{array}{ll} \frac{d}{dx}\sinh(x) = \cosh(x) & \qquad \frac{d}{dx}\cosh(x) = \sinh(x) \\ \sinh(0) = 0 & \qquad \cosh(0) = 1 \end{array}$$

Use the relationship above to determine the Maclaurin series for $\sinh(x)$ and $\cosh(x)$.
Hint: If $f(x) = \cosh(x)$, the above facts tell you the value of all the derivatives $f^{(n)}(0)$.

2. Using the series for $\cosh(x)$ and $\sinh(x)$ and the fact that $i^2 = -1$, verify that

- $\cosh(x) = \frac{1}{2}\left(e^x + e^{-x}\right)$
- $\sinh(x) = \frac{1}{2}\left(e^x - e^{-x}\right)$
- $\cosh(x) = \cos(ix)$
- $\sinh(x) = -i\sin(ix)$