MAT132, Paper Homework 6

1. The Bessel function of order 0 is given by

$$J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$$

which converges for all x. Bessel functions have many applications, for example in describing the motion of a vibrating drumhead.

Calculate $J_0(1)$ to three places.

~~a~~) Show that $J_0(x)$ satisfies the differential equation $x^2 J_0''(x) + x J_0'(x) + x^2 J_0(x) = 0$.

2. Calculate the Maclaurin series for $f(x) = \cos(x^2)$.