

### Question 1 rubric

Before contacting the instructor about your score, please REVIEW the RUBRIC CAREFULLY. You will probably understand why points were taken off. If you have any concerns, please contact the instructor who graded this question for all students (see contact information at the end of the rubric).

A correct solution should have the following elements, with partial credit given as follows:

Correct answer: "No, it does not always converge" or "it can diverge": 2pts

A plot is included, showing a sequence going to  $-\infty$ : 6pts

The plot is accompanied with

- EITHER an explicit formula (like  $a_n = -n$  or  $x_n = -n^2$ ) OR some explanation/statement for divergence to  $-\infty$  (like saying that sequence "decreases indefinitely and goes to *infy*", or similar words to that effect)

- AND some evidence of an upper bound (explicit number/statement, or simply a horizontal line drawn above the sequence)

*Note:* no formula was required if there's sufficient graphical explanation/some words, and no work was needed to show upper bound+decreasing for "obvious" sequences.

Correct answer with explanation but no plot: 9pt

Partial credit: 1 pt for answer "No"; 1pt for statements "increasing sequence, bounded above: converges" and/or "decreasing, bounded below: converges"

Points were taken off for the following mistakes:

statements "it diverges" or "it does not converge" (in the situation described in the question, convergence is *possible* but not guaranteed): -0.5pt

no distinction between a function and a sequence: all work is done for a function, -3pts; work done for a sequence but graph of function is given, -1pt.

Question 3 was graded by Hugo Mainguy, hugo.mainguy@stonybrook.edu. He can be contacted with any concerns about your score for this question.

Before contacting the instructor about your score, please REVIEW the RUBRIC CAREFULLY. You will probably understand why points were taken off. If you have any concerns, please contact the instructor who graded this question for all students (see contact information at the end of the rubric).

Question 2 Rubric:

- (a) 3pts
- (b) 2pts
- (c) 2pts
- (d) 3pts

(a) Correct, full solution:

Divide top and bottom by  $n^2$   
 Use limit laws  
 Compute the limit of each term on top and bottom  
 Correct result  
 or  
 Used L'Hopital's Rule several times with justification

Wrong Result Skipped Steps 0pt

Found the sequence to diverge, no steps

Skipped Steps .5pt

Wrote answer without any/valid justification

Skipped Steps 1pt

Cited ratio of highest powers as answer

Skipped Steps 1.5pt

Clearly explained insignificance of lower order terms in the limit

Did not divide top and bottom by  $n^2$

Did not cite limit laws

or

Used L'Hopital's Rule without justification

Skipped Steps 2pt

Clearly explained insignificance of lower order terms in the limit

Did not divide top and bottom by  $n^2$

Used limit laws in explanation

Falsely Declare Divergence 2pt

Correct Process and calculation of limit

Declare result to diverge anyway

Skipped Steps 2.5pt

Correct process, but did not mention or explicitly use limit laws

Correct result

Correct Process, Computation Error 2.5pt

Made a computational error, process otherwise ok

or

Correct process, significant notation problems leading to false equations

(b) Correct, full solution:

Explain relationship between limit of partial sums and sum of series

Same result as previous step

Did not realize that  $S_n$  is the partial sum of the series 0pt

Tries to compute the sum of the series of  $S_n$  from the previous part

Recalculates Result 1pt

Gets result by repeating calculation of previous part

No explanation 1.5pt

Gives correct answer, no explanation at all

(c)

Correct, full solution:  
 Writes equations for  $S_1$  and  $S_2$  as partial sums of series elements  
 Solve for first series element equal to  $S_1$   
 Solve for second series element equal to  $S_2 - S_1$

Confused  $S_n$  with original series 0pt  
 Wrote  $S_1$  as first term of series and  $S_2$  as second  
 Added partial sums together 0.5pt  
 Obtained results as  $S_1$  and  $S_1 + S_2$   
 Not recognizing  $S_n$  as partial sums

No Explanation .5  
 Simply wrote correct answer with no explanation

Right process, wrong implementation 1pt  
 Recognized  $S_n$  as partial sums  
 Wrote equations incorrectly, then solved

Major computational error 1pt  
 Correct equations and process  
 Several or severe arithmetic mistakes in result

Minor computational error 1.5pt  
 Correct equations and process  
 Arithmetic mistake in result

(d)

Correct, full solution:  
 Recognize that  $S_n$  is the sequence of partial sums  
 Use convergence of the series to conclude that conditions of Divergence Test must not apply, so sequence cannot diverge or converge to anything other than 0  
 Conclude that the sequence must converge to 0

Applied Divergence Test incorrectly 0pt  
 Misunderstands conditions or results of divergence test  
 Incorrect result

No justification 0pt  
 Wrote "Converges" or "Diverges" without any explanation

Used Divergence Test on  $S_n$  0.5pt  
 Applied divergence test to conclude that the sum of  $S_n$  diverges

Mistook formula for  $S_n$  as formula for term of sequence .5pt  
 Confuses the equation for  $S_n$  as the equation for the series  
 Found the sequence to converge to the limit of  $S_n$

Correct Result without explanation 1pt  
 Found the sequence goes to 0, no explanation

Correct Result partial explanation without mentioning Divergence Test 2pt  
 Found the sequence goes to 0, wrote explanation without the Divergence Test

Correct Result minimal explanation 2pt  
 Found the sequence goes to 0, wrote "divergence test" and nothing further

Question 2 was graded by Matthew Dannenberg, matthew.dannenberg@stonybrook.edu. He can be contacted with any concerns about your score for this question.

### Question 3 rubric

Before contacting the instructor about your score, please REVIEW the RUBRIC CAREFULLY. You will probably understand why points were taken off. If you have any concerns, please contact the instructor who graded this question for all students (see contact information at the end of the rubric).

(a) 4pts, (b) 3pts, (c) 3pts

(a)

Partial credit level 1: Series identified as geometric series, "converges for  $|ratio| < 1$ " stated, no further work (plot shows interval  $(-1,1)$  or not present): 1pt

Partial credit level 2: Series identified as geometric series, "converges for  $|ratio| < 1$ " stated, an attempt to compute ratio given but ratio incorrect, no plot given: 2pts

Partial credit level 3: same as above, plot given (plot may be incorrect but \*consistent\* with the ratio calculations): 3pts

Correct and complete solution: Series identified as geometric series, "converges for  $|ratio| < 1$ " stated, ratio computed correctly, plot is correct.

Everything is correct but no plot given: lose .05pt

For all levels: lose .5 point if ratio=1 given as convergent (no penalty if  $|r| < 1$  but endpoints included on the plot)

Note: some students used ratio test. This test wasn't covered in class yet and shouldn't be used for geometric series. Correct applications of ratio test receive 1 pt.

(b) NOT GRADED ON THE NUMBER OF CORRECT ANSWERS.

Partial credit level 0.5: statement that series converges for  $|r| < 1$ , further work inconsistent: 0.5pts

Partial credit level 1: converges at 0, 1pt

Partial credit level 2: converges for  $|ratio| < 1$ , ratio incorrectly identified but work is consistent;  $c = 0$  marked as "convergent" 2pts (1.5 pts only if the value of the parameter is taken to be the ratio),

Partial credit level 2.5: ratio correctly identified, "converges for  $|ratio| < 1$ " stated, given numbers compared to the ratio but with small numerical mistakes or  $r = 1$  or  $r = -1$  incorrectly labeled as convergent, 2.5pts

Correct and complete solution: ratio correctly identified, "converges for  $|ratio| < 1$ " stated, all answers correct, 3pts

Note: Due to instructors' oversight, the version of the question in 127.02 included a series that is undefined for  $l = 0$ . (This issue would not arise if the summation started with  $n = 1$  or if the numerator had exponent  $(n + 1)$  instead of  $(n - 1)$ .) Most students overlooked this issue as well, stating that the series converges for  $l = 0$ . Such answers were given credit; the few students who noticed the issue were given 0.5 bonus points.

(c)

Partial credit level 1: using the sum formula for geometric series, but incorrect ratio and first term and/or bad value chosen: 1pt

Partial credit level 2: correct manipulations with  $a + ar + ar^2 + \dots = \frac{a}{1-r}$  formula, some mistake with first term or ratio: 2pt

Partial credit level 3: appropriate parameter value chosen, correct application of the sum formula, small arithmetic mistake in the final answer: 2.5pts

Correct and complete solution: correct application of the sum formula for an appropriate parameter value, correct answer: 3 pts

Question 3 was graded by Runjie Hu, runjie.hu@stonybrook.edu. He can be contacted with any concerns about your score for this question.

### Question 4 rubric

Before contacting the instructor about your score, please REVIEW the RUBRIC CAREFULLY. You will probably understand why points were taken off. If you have any concerns, please contact the instructor who graded this question for all students (see contact information at the end of the rubric).

Incorrect steps are B (2pts), C(2pts), E(3pts), F(3pts)

In A, the function is continuous and positive, and decreasing on  $(1, +\infty)$ , as required by the Integral test. A few students pointed out that the interval where  $f$  decreases is not stated in the solution, and that  $f$  increases from 0 to about 0.5. A small extra credit was awarded for this observation.

**B:** mistake in integration,  $du = 2xdx$ , not  $xdx$ , another mistake with antiderivative of  $\frac{1}{u^2}$ ; Hermione's answer for the antiderivative is actually correct when mistakes are combined.

Partial credit: Spotting mistake, not providing correct calculation: 1pt

Complete solution: Spotting mistake +providing correction: 2pts

**C:** cannot plug in infinity into antiderivatives for improper integrals, must work with limits of integrals over finite intervals

Partial credit: spotting that infinity shouldn't be plugged in but not providing correct calculation: 1pt

Complete solution: Spotting mistake +providing correction: 2pts

**E:** two mistakes

(1) Hermione draws boxes on half-integers, the width of the boxes is 0.5, which gives  $0.5(\frac{1}{(1^2+1)^2} + \frac{1.5}{(1.5^2+1)^2} + \frac{2}{(2^2+1)^2} + \frac{2.5}{(2.5^2+1)^2} + \dots)$ , not the terms of the series

(2) Boxes should go under the graph when working with convergence

Partial Credit - level 1: spotted (2) but not (1), made picture on half-integers repeating mistake (1): 1 pt

Partial Credit - level 2: spotted (2) but not (1), explained that boxes must be inside or drew unlabeled boxes under graph: 1.5 pt

Partial Credit - level 3: spotted mistake (2), didn't point out mistake (1), but made a correctly labeled picture: 2.5 pts

Complete solution: spotting both mistakes, providing correction: 3pts

**F:** The given Riemann sums are not a close approximation of the integral in this case, so Hermione's argument gives no conclusion.

Correct answer: when boxes are drawn under graph, the series is represented by the total area of boxes, *SMALLER* than the area under graph. Area under graph=integral=finite number since the integral converges, so series has finite sum and is convergent.

Partial credit: if mistake pointed out but explanation vague or unsatisfactory: 1 or 2pts

Complete solution: must clearly state that the area of rectangles representing series is *SMALLER* than the finite value of the improper integral, 3pts

Question 4 was graded by Olga Plamenevskaya, [olga.plamenevskaya@stonybrook.edu](mailto:olga.plamenevskaya@stonybrook.edu). She can be contacted with any concerns about your score for this question.