Midterm 1
MAT 127
Oct 1, 2015

Name:		ID #:
(please print)		
Your section:	(see list below)	

Lecture 01	MWF 10:00 AM – 10:53 AM	Alexander Kirillov
Lecture 02	MW 5:30 PM – 6:50 PM	Mark McLean
Lecture 04	TUTH 5:30 PM – 6:50 PM	Sabyasachi Mukherjee

No notes, books or calculators.

You must show your reasoning, not just the answer. Answers without justification will get only partial credit. Your solutions should be written so that the grader is able to follow your reasoning and computations.

Please cross out anything that is not part of your solution — e.g., some preliminary computations that you didn't need. Everything not crossed out will be considered part of your solution and graded.

When computing numerical answers, please do not replace algebraic expressions and constants such as $\sqrt{2}$ or π by approximate values — instead, leave $\sqrt{2}$ in your final answer.

	1	2	3	4	5	**Total**
	15pt	15pt	20pt	10pt	10pt	70pts
Grade						

1. *(15 pts)*

(a) Determine whether the following sequence converges. If it converges, find the limit

$$a_n = (-1)^n \frac{n^3 + n + 4}{n^3 - 2n^2 + 4}$$

(b) Determine whether the following sequence converges. If it converges, find the limit

$$a_n = \frac{n^2 + 3}{(2n - 1)3^n}$$

(c) Consider the sequence given by $a_1 = \sqrt{2}$, $a_2 = \sqrt{2a_1} = \sqrt{2\sqrt{2}}$, $a_3 = \sqrt{2a_2}$, ..., $a_{n+1} = \sqrt{2a_n}$. It is known that this sequence is monotonically increasing. Explain why this sequence must be convergent, and find the limit.

2. *(15 pts)*

(a) Determine whether the following series converges or diverges

$$\sum_{n=0}^{\infty}(-1)^n\frac{n}{n^2+1}$$

(b) Determine whether the following series converges or diverges

$$\sum_{n=0}^{\infty}\frac{n^3+1}{n^4+2n+3}$$

(c) Determine whether the following series converges or diverges

$$\sum_{n=1}^{\infty} \frac{1}{n(\ln(n))^2}$$

3. *(20pts)*

(a) Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}$.

(Hint: factor the denominator!)

(b) Write the number $2.1\overline{79} = 2.1797979\ldots$ as a fraction

4. *(10 pts)*

Find the radius of convergence, and interval of convergence of the power series:

$$\sum_{n=1}^{\infty} \frac{4^n (x-2)^n}{\sqrt{4n+1}}.$$

You are not required to determine whether the series is convergent at the endpoints of the interval of convergence.

5. *(10 pts)*

Write the following function as a power series in x. Determine the radius of convergence of this series.

$$f(x) = \frac{x}{3x^2 + 1}$$