EXERCISE ONE Scientists put 1000 bacteria in a container which can sustain at most 10000 bacteria. Assume that the number of bacteria is modeled by the logistic equation

$$P' = rP(K - P).$$

After 5 hours, there are 5000 bacteria in the container. How many bacteria can we find in the container after 7 hours? (7 points)
If we put 20000 bacteria in the same container and we assume that they are modeled by the same logistic equation, then what is roughly the poplulation of bacteria after a month? (3 points)
The carrying capacity is $K = 10000$. The solution of the logistic equation with initial condition $P(0) = 1000$ is $P(t) = \frac{1000e^{rt}}{0.9+0.1e^{rt}}$. The poplulation after 5 hours is $5000 = \frac{1000e^{5r}}{0.9+0.1e^{5r}}$. This implies $e^{5r} = 9$ and thus $r = \frac{1}{5}\ln(9) \approx 0.1908485018$. Then $P(7) = \frac{1000e^{7r}}{0.9+0.1e^{7r}} = 7065.9211397$.
Since 20000 is larger than the carrying capacity, the solution with initial condition $P(0) = 20000$ is decreasing and converges to K when the time goes to infinity. A month is a very long period with respect to our scale, thus after a month the population is approximately just the carrying capacity 10000.