EXERCISE ONE Find the solution of the differential equation

$$y' + 3 = 2y$$

with initial condition $y(0) = 2$.

We rewrite the equation as $y'/(2y-3) = 1$. The function $\frac{1}{2}\ln|2y-3|$ is an antiderivative of $y'/(2y-3)$ (as a function of x). The antiderivatives of the constant function 1 are $x+C$ where C is an arbitrary real number. Therefore $\frac{1}{2}\ln|2y-3| = x+C$ for some C. Taking exponentials of both sides, we get $2y = 3+e^{x+C}$ or $2y = 3-e^{x+C}$. As e^{x+C} is always positive, the initial condition $y(0)=2$ is not possible in the second case. Thus $2y = 3+e^{x+C}$. The initial condition $y(0)=2$ can be now written as $2\times 2 = 3+e^{C}$. This implies that $C=0$. Thus the desired solution is $y = \frac{1}{2}(3+e^{x})$.

EXERCISE TWO Find the solution of the differential equation

$$y^2 y' = 2x$$

with initial condition $y(1) = 2$.

The function $\frac{1}{3}y^3$ is an antiderivative of $y^2 y'$ (as a function of x). The antiderivatives of $2x$ are $x^2 + C$ where C is an arbitrary real number. Therefore $\frac{1}{3}y^3 = x^2 + C$ for some C. This can be written as $y = (3x^2 + 3C)^{1/3}$. The initial condition $y(1) = 2$ can be now written as $2 = (3 + 3C)^{1/3}$, namely $8 = 3 + 3C$. Thus $3C = 5$ and the desired solution is $y = (3x^2 + 5)^{1/3}$.