- FIND THE MACLAURIN SERIES FOR $(1+x^2)^{3/2} + 3\cos(x)$.

WHILE WE CAN DO THIS BY TAKING DERIVATIVES, ETC IT IS QUICKEST AND EASIEST TO USE KNOWN SERIES.

FROM THE BINOMIAL SERIES

$$(1+x)^{3/2} = \sum_{n=0}^{\infty} \binom{3/2}{n} x^n = 1 + \frac{3}{2}x + \frac{\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)}{2}x^2 + \frac{\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)}{3!}x^3 + \cdots$$

AND WE KNOW THE SERIES FOR $\cos(x)$ AS

$$\cos(x) = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

SO

$$(1+x^2)^{3/2} + 3\cos(x) = \overbrace{(1+3)}^{4} + \overbrace{\left(\frac{3}{2}x^2 - 3\frac{x^2}{2}\right)}^{\text{ZERO}} + \underbrace{\left(\frac{3}{8}x^4 + \frac{3x^4}{4!}\right)}_{x^4/2} - \underbrace{\left(\frac{x^6}{8} + \frac{3x^6}{6!}\right)}_{\text{DON'T CARE}} \cdots$$

$$= \sum_{n=0}^{\infty} \left(\binom{3/2}{n} + \frac{3}{(2n)!} \right) x^{2n}$$

USE THIS TO EVALUATE $\displaystyle\lim_{x\to 0} \frac{(1+x^2)^{3/2} + 3\cos(x) - 4}{x^4}$

JUST REPLACE WITH TERMS OF THE SERIES AND BE HAPPY:

$$\lim_{x\to 0} \frac{\left(4 + \frac{x^4}{2} - \left(\frac{1}{8} + \frac{3}{6!}\right)x^6 + \cdots\right) - 4}{x^4} = \lim_{x\to 0} \frac{\frac{x^4}{2} - \left(\frac{1}{8} + \frac{3}{6!}\right)x^6 + \cdots}{x^4}$$

$$= \lim_{x\to 0} \left(\frac{1}{2} \overbrace{- \left(\frac{1}{8} + \frac{3}{6!}\right)x^2 + \cdots}^{\text{HIGHER POWERS OF } x} \right) = \frac{1}{2}$$

IF WE USED L'HOPITAL'S RULE, WE'D HAVE TO TAKE DERIVATIVES 4 TIMES ☹